

# Direct hybrid Navigation based on celestial bodies and landmarks

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# Agenda

- ◆ The "New navigation" initiated by A. YVON-VILLARCEAU (1877)
- ◆ One century later...
- ◆ Sight reduction through Cones for land & sea applications
- ◆ Measurement functions for multi-sensor integrating:
  - Stars, Satellites & High Altitude Platforms, Landmarks
  - from loose coupling to tight coupling
- ◆ Unified Matrix equation for direct hybridation
- ◆ A canonic case with Stars, Satellites and both
- ◆ A practical case with 2 Landmarks and a Star
- ◆ Outline of an Architecture for Autonomous Vehicles
- ◆ Conclusion & References



# YVON-VILLARCEAU, pioneer of modern linear methods

- ◆  $e = a x + b y + c z$       [  $e = \sin h$  ]
- ◆  $e' = a' x + b' y + c' z$       [  $e' = \sin h'$  ]
- ◆  $1 = x^2 + y^2 + z^2$

*where  $h$  &  $h'$  are the heights of 2 celestial bodies*



"x, y, z represent rectangular coordinates of the location, according to a linear scale, with the Earth radius taken as unity (=1). So the 2 first equations generate **2 planes**, and the last equation the **Earth sphere itself**. Therefore, the problem is reduced to compute the intersection of **2 planes** with **a sphere**.

But in practice, we don't recommend to use the given formulas, because of the associated computing difficulties **[on 1877!]**. We propose a method which is **LESS DIRECT**, but quicker to be applied. "

*« Nouvelle Navigation Astronomique – Théorie » pp 22-26 (Gauthier-Villars, Paris, **1877**)*

*Marcq Saint-Hilaire method, associated errors and their mathematics were following on.*



# In 1877, there were alternative methods for a long cruise!

James Tissot  
1877



*Leaving Portsmouth (UK), "from the gallery of HMS Calcutta"*



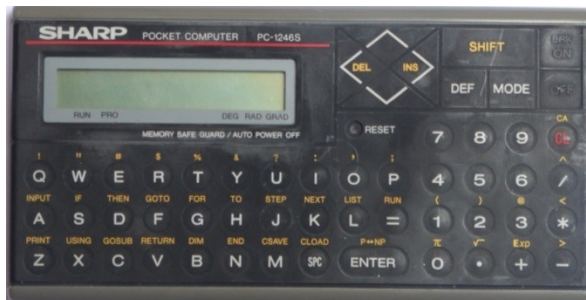
# YVON-VILLARCEAU followers ...?

- ◆  $e_1 = a_1 x + b_1 y + c_1 z$      $[e_1 = \sin h_1]$
- ◆  $e_2 = a_2 x + b_2 y + c_2 z$      $[e_2 = \sin h_2]$
- ◆ .....  
.....
- ◆  $e_n = a_n x + b_n y + c_n z$      $[e_n = \sin h_n]$
- ◆  $(1 = x^2 + y^2 + z^2)$  , skipped if  $n \geq 3$

*where  $h_k$  is the height of celestial body  $k$  ( $= 1...n$ )*



**One century** has been spent before using these equations...  
... firstly for **Celestial navigation:**



with HP 25, HP 97 (ENMM, 1976), TI 59 (USNO, 1978)...  
then SHARP 1246S & BASIC pocket calculators (1984)



# Using CelNav for topography along Sahara coast (1997)

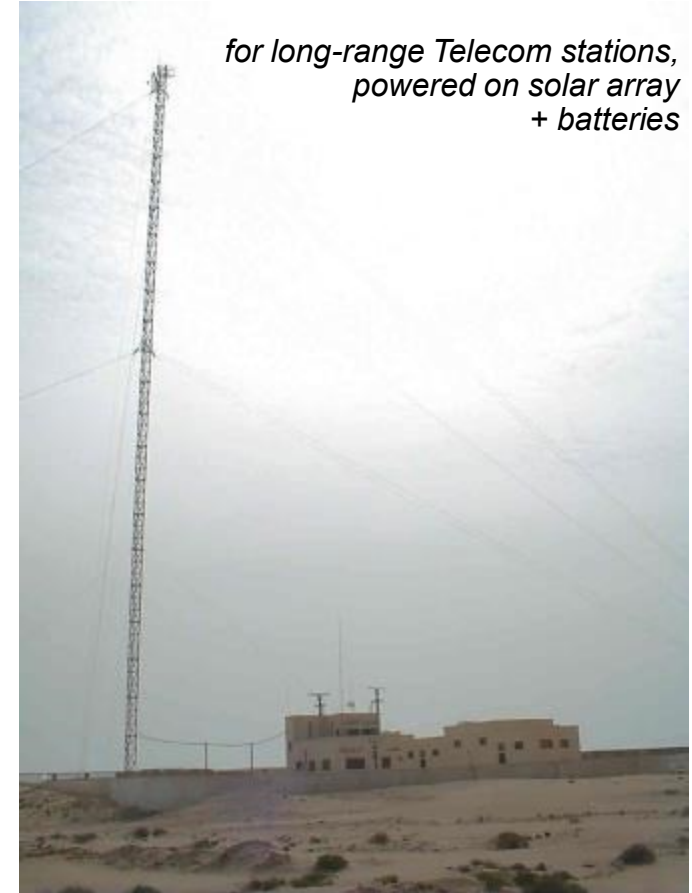
## ◆ Doing some SLAM with better accuracy than GPS under SA !



- SOKKIA DT5, **connected** as a digital theodolite to...



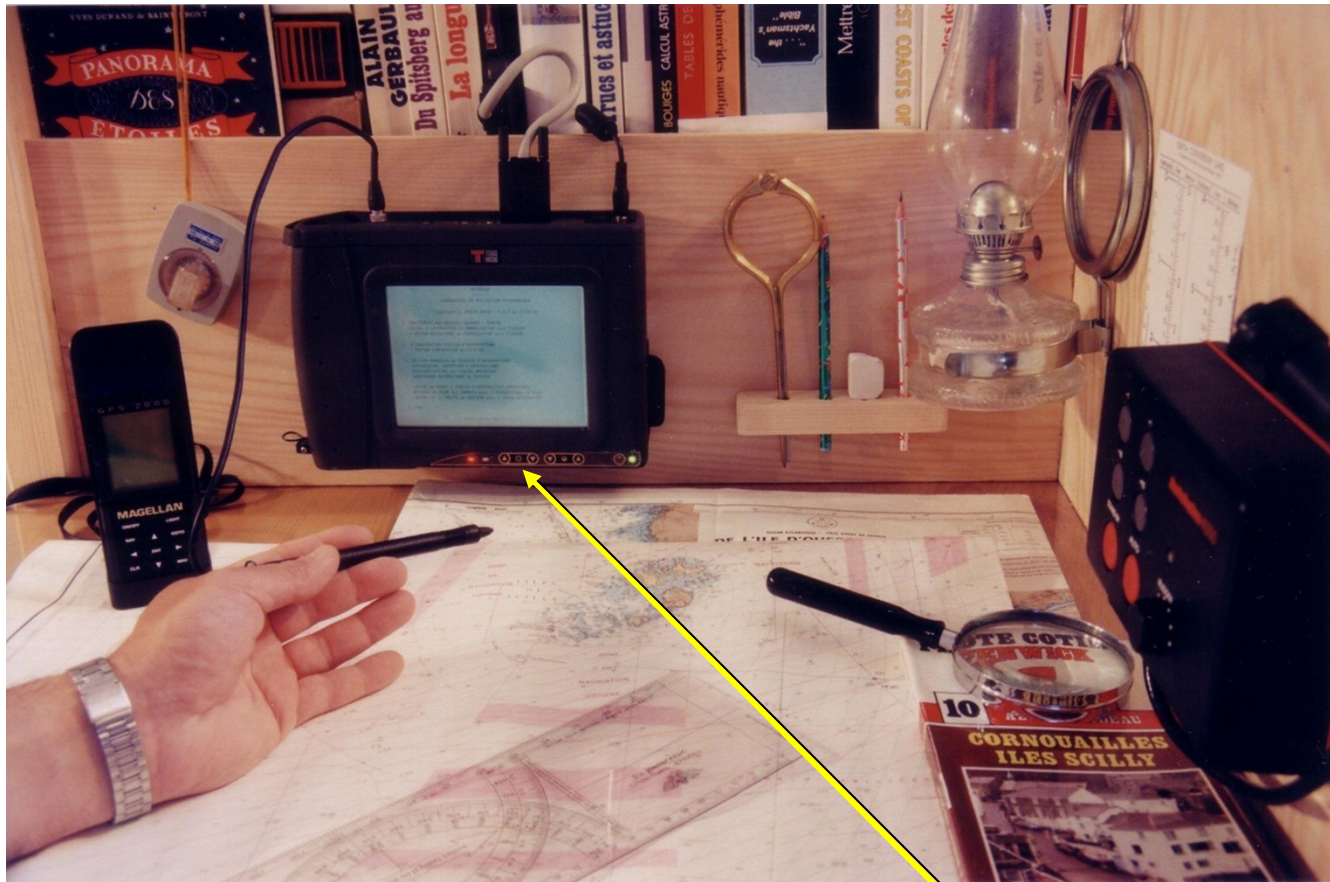
- ...HP 200LX, palm PC running automatic ephemerids + multi-sight reduction



*for long-range Telecom stations,  
powered on solar array  
+ batteries*



# Using CelNav & GPS on board a 36 footer (1998)



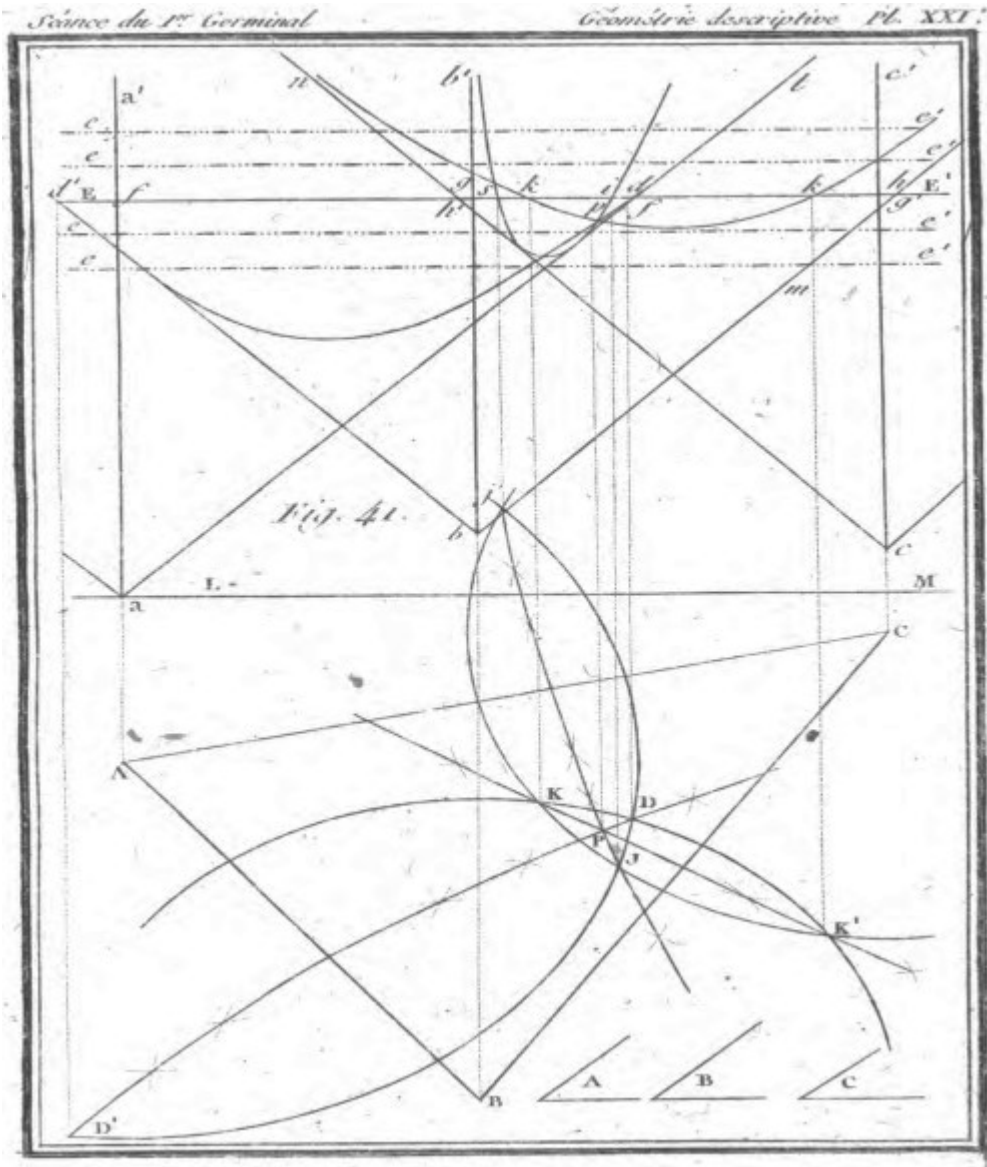
*Jeanneau Sun-Odysey*



- **MAGELLAN GPS 2000, an early handheld RX**
- **Ruggedized PC running DOS, and ASTROLAB software**



# Monge Descriptive Geometry and positioning by cones



- ◆ Can you **find the projections of a point** when you know the zenith angles of 3 marks in space ?
- ◆ The position is given by intersecting **3 circular cones**
- ◆ (Then Monge proposed a similar approach with multilateration !)
- ◆ Séances des Ecoles Normales,  
*Paris, Imprimerie du Cercle Social,  
Rue du Théâtre Français, N°4*  
French Republic Germinal 1 an IV  
(March 21, 1796)



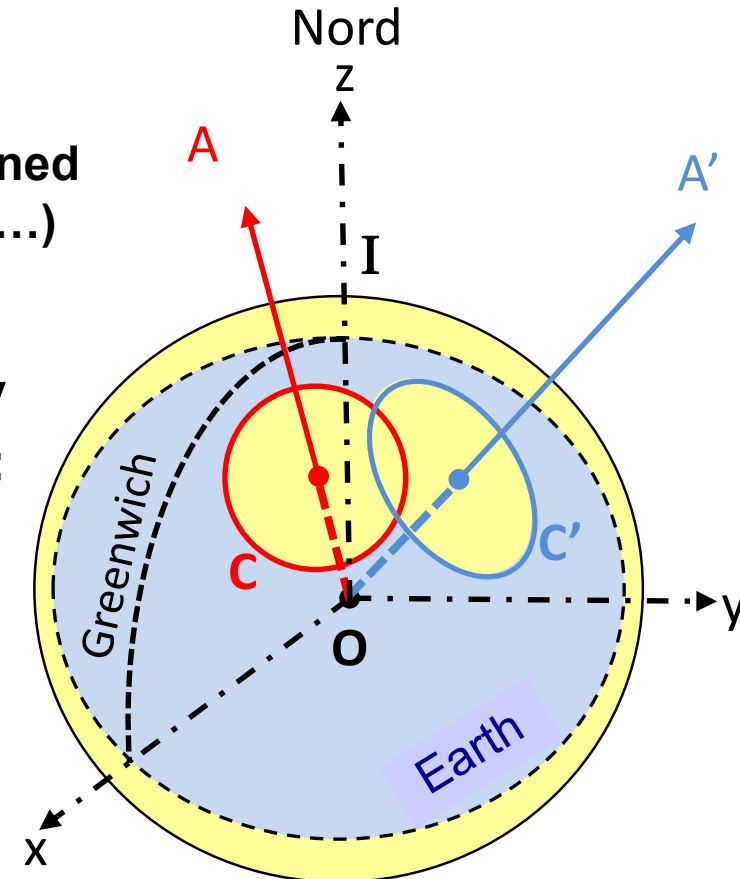
# Geometry basis for a multi-sensor integration (1/2)

## ◆ Around the **Earth**, a local reference sphere can be considered:

- this is "spherical Earth", with radius = 1 for Celestial (and coastal) Navigation
- but it can be a **virtual sphere**, with a radius  $I$  to be defined for Navigation using Satellites or HAP (UAV, balloons...)  $I$  remains near 1 in conventional aviation airspace

## ◆ Such a reference sphere gives a circular geometry to locus of points with an equal measure of a mark:

- around this mark or its vertical projection on the sphere
- locus is a **small circle C** of the sphere,
- circle C is centered in the **OA** axis to a celestial mark
- it may be centered or not to a group of landmarks (if not, the shift vector is known)



*ECEF coordinate system  
Earth Centered, Earth Fixed*




# Geometry basis for a multi-sensor integration (2/2)

## ◆ The envelope of horizontal planes following a **circle C** is a cone:

- with the same axis OA than the circle C
- tangent to the sphere I by definition

## ◆ The **vertex S** of the cone is:

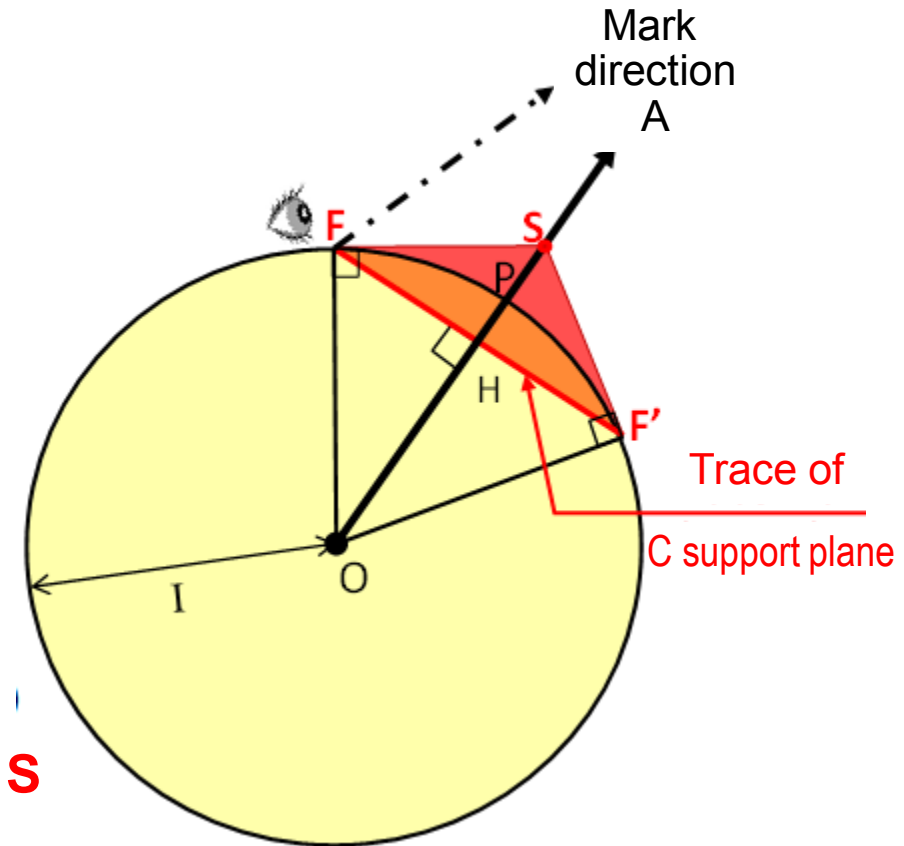
- in the horizon plane of the observer (if measure is exact) 
- representative of the mark via its 2D location (P on the sphere)
- representative of the measure via its altitude

## ◆ **OS** radius is directly **tied to the measure** of:

- time or distance (satellite, HAP,...Radar)
- angle (star, height or aperture between landmarks)

## ◆ In every case, marks can be exchanged with **S** in the horizon plane of the observer

- celestial bodies are **lowered** (as with a sextant)

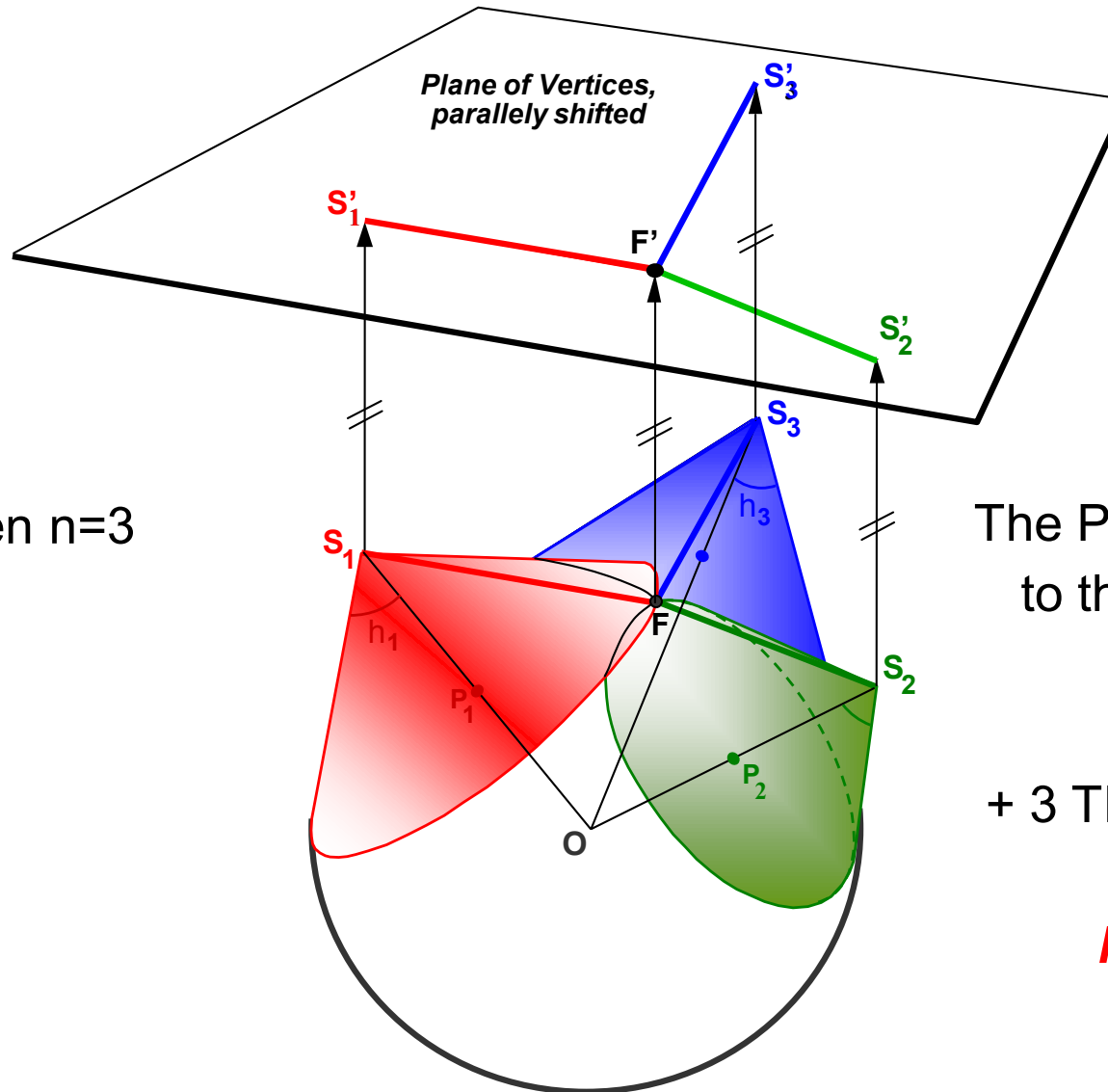


*Diametral cross view*



# $n$ measures with associated cones $\Rightarrow$ Plane of Vertices

Case when  $n=3$



## THEOREM 1

The Plane of Vertices is tangent to the reference sphere in  $F$  for  $n = 3$  and  $\forall n$

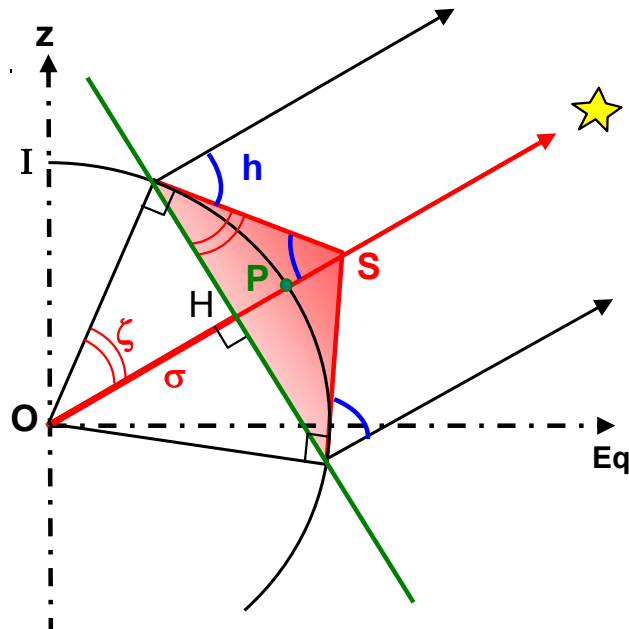
+ 3 THEOREMS in succession

**No Need for any AP**  
(Assumed Position)



# How to decline it with respect to the type of mark (1/2)

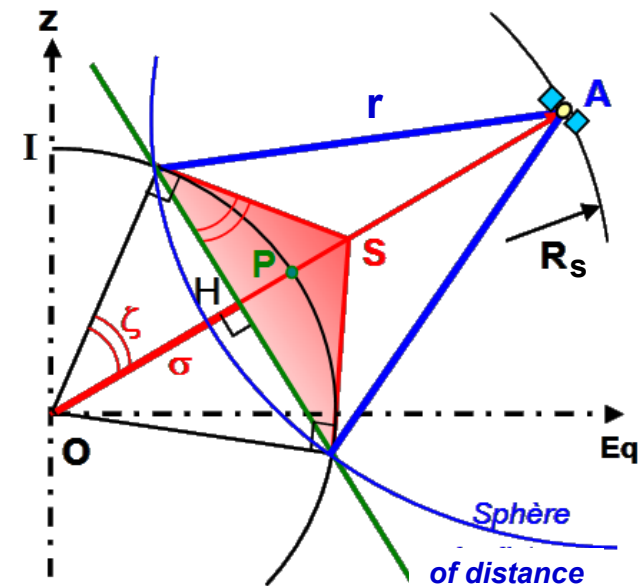
- ◆ Case ① with a Star
  - measured variable:  $h$



$$\Sigma = \overline{OS} = \frac{I}{\sinh} \quad \text{where } I = 1$$

$$= I \cdot \operatorname{cosec} h$$

- ◆ Case ② with a Satellite (or HAP)
  - measured variable:  $r$



$$\Sigma = \overline{OS} = \frac{2 \cdot R_s}{1 + \frac{R_s^2 - r^2}{I^2}} \quad \text{where } r = \rho + c \cdot \tau$$

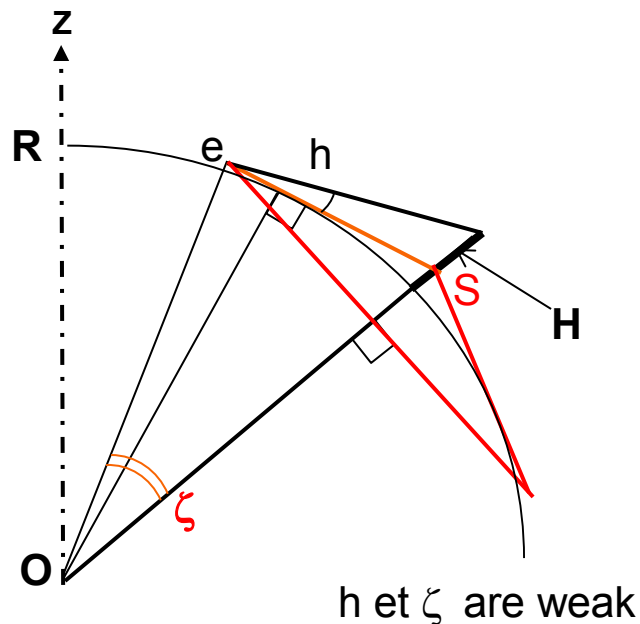
$\zeta$  = zenith distance



# How to decline it with respect to the type of mark (2/2)

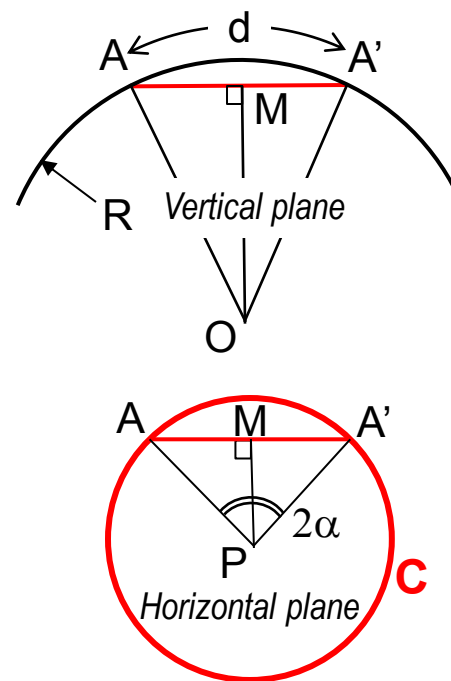
## ◆ Case ③ with an isolated Landmark

- measured variable:  $h$



## ◆ Case ④ with a pair of Landmarks A, A'

- measured variable:  $\alpha$



M: middle point of AA'

$$\sin \zeta = \frac{d}{2} \cdot \frac{1}{\sin \alpha}$$

$\zeta$  weak except if  $\alpha$  is weak also

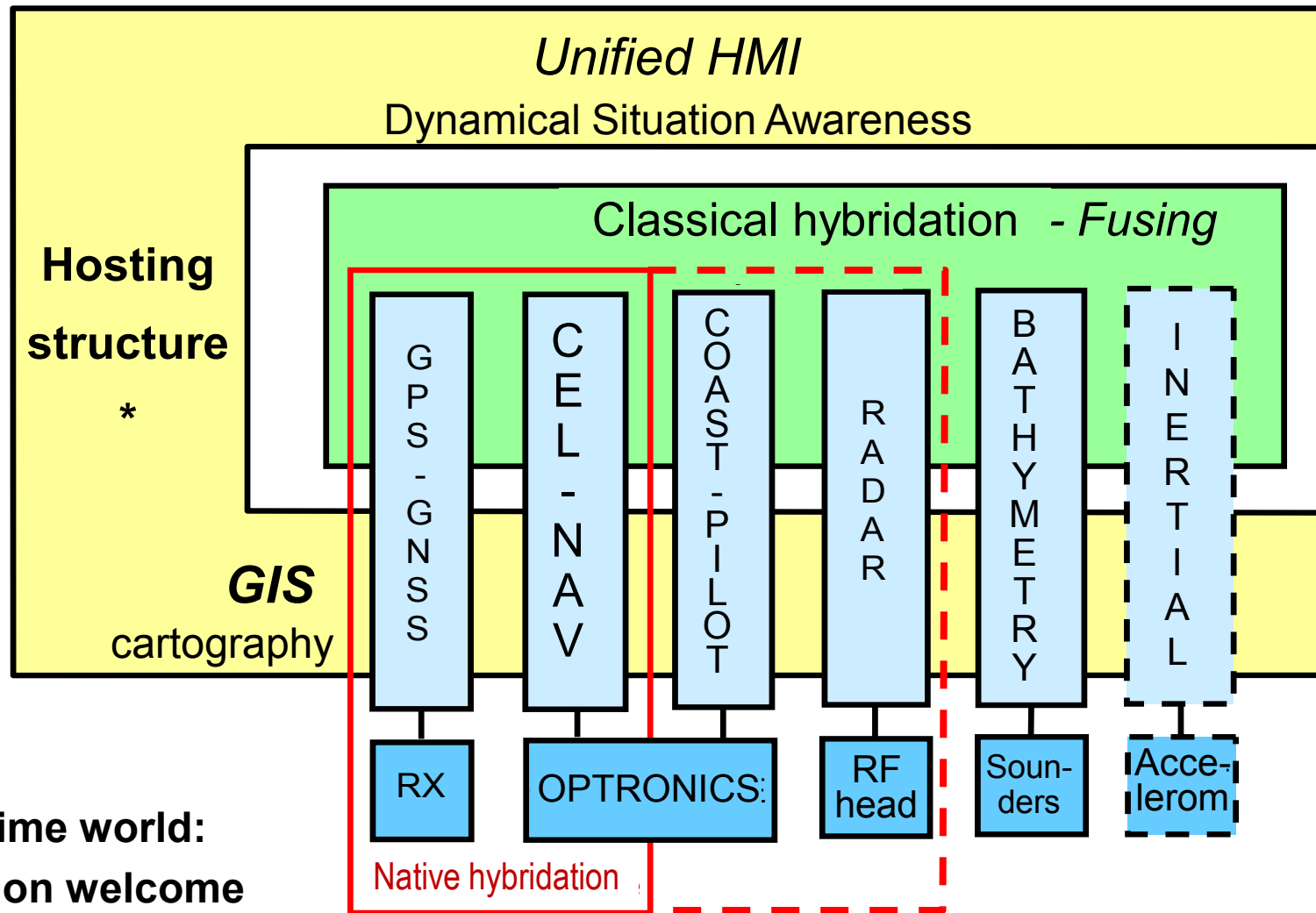
$$\Sigma = 1 / \cos \zeta$$

P is the center of the arc of **C** which subtends  $\alpha$

For P location computing return to case ①



# Principle of multi-sensor Hybrid Navigation

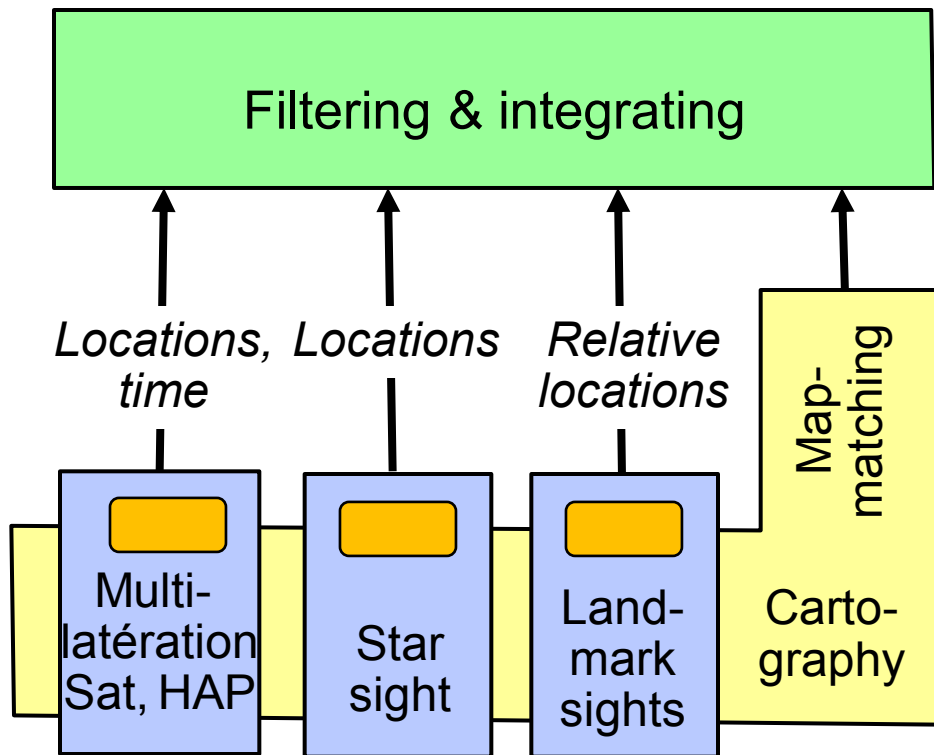


\* In maritime world:  
ECDIS station welcome

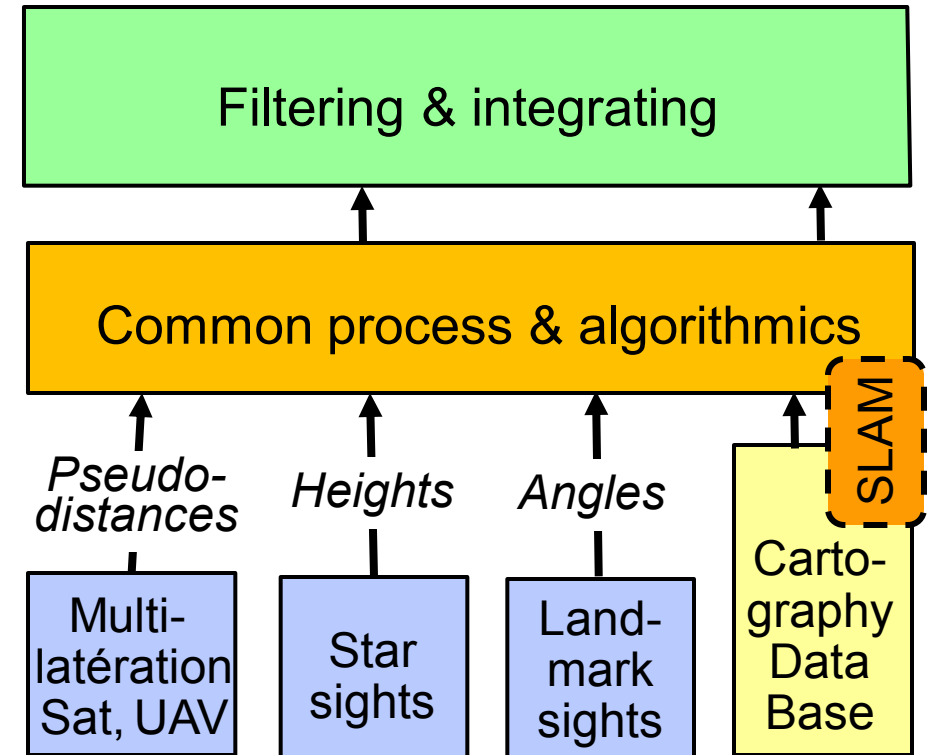


# From loose coupling to tight coupling

- ◆ Fusing **pre-processed** data or **raw** data (which is better)



■ Loose coupling



■ Tight coupling

(SLAM = Simultaneous Localization & Mapping)



# A single and polyvalent matrix equation using Vertices

$$\begin{array}{c} \text{Vertices Matrix} \end{array} \begin{bmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ \dots & \dots & \dots \\ X_i & Y_i & Z_i \\ \dots & \dots & \dots \\ X_m & Y_m & Z_m \\ X_n & Y_n & Z_n \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} I \\ I \\ \dots \\ 1 \\ \dots \\ 1 \\ 1 \end{bmatrix} \begin{array}{l} \text{Slice for Satellites} \\ \text{\& HAP} \\ \\ \text{Slice for Stars} \\ \\ \text{Slice for landmarks} \end{array}$$

- ◆ **Size of Vertices matrix:  $n \times 3$** 
  - 3D coordinates of  $n$  **Vertices** (including their  $\Sigma_k$ ) are distributed according to lines
- ◆ **Without satellites (or HAP)  $\Rightarrow$  pseudo-inversing the Vertices matrix**
  - pseudo-inversing is equivalent to LMS solving
- ◆ **With satellites (or HAP): favor is set to horizontal (2D) coordinates**
  - $\Sigma_k$  are parametrized with **altitude**  $I$  and **time**  $\tau \Rightarrow$  **iterative process** is necessary
  - Solving through **relaxation** : pseudo-inversing **once**,  $I$  &  $\tau$  matched at each iteration or **incrementation** : pseudo-inversing at each iteration

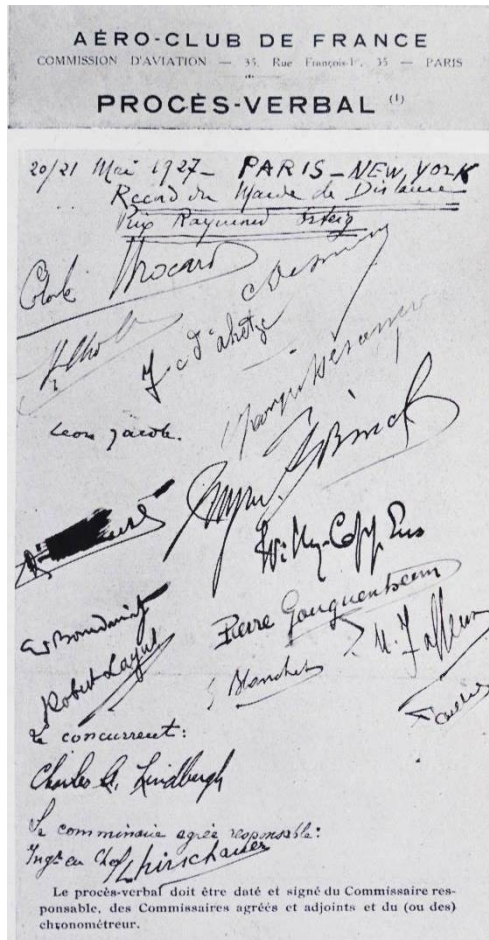


# Minimum conditions about the number of marks

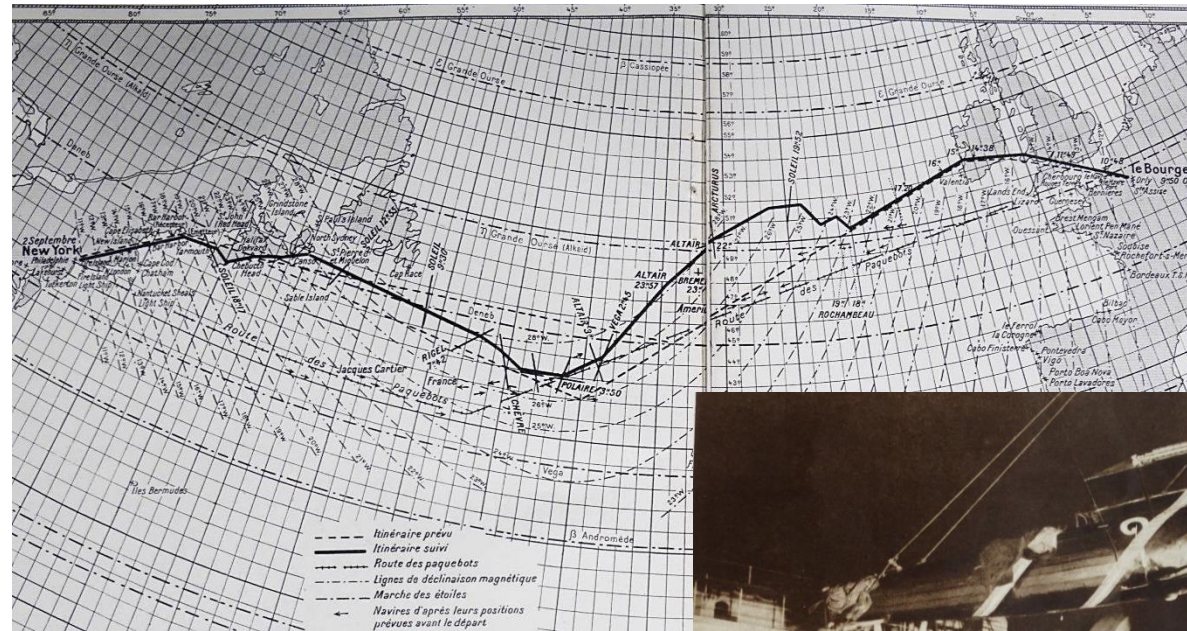
- ◆ **Satellites only:**
  - 4 for 3D location (aviation)
  - 3 for 2D location (on ground or at sea)
- ◆ **Satellites + Stars (or planets):**
  - 2 Satellites : i.e. the 2 GEO of new generation WAAS...
  - and 1 star : Sun, Moon or Venus
- ◆ **Star and Landmark: 1 of each for a landing from high sea**
- ◆ **Satellites and Landmarks: sea straights and urban canyons**
  - 2 Satellites + 2 Landmarks (or urban objects)
- ◆ **Landmarks only:**
  - 2 to 3 coastal landmarks
  - 3 urban objects



## 3 pages of Navigation history... for a canonic case



- **French certificate to Lindbergh, 1927**



- **Atlantic record of « ? »**  
**Paris- NY, Costes & Bellonte,**  
**Sept 1930**

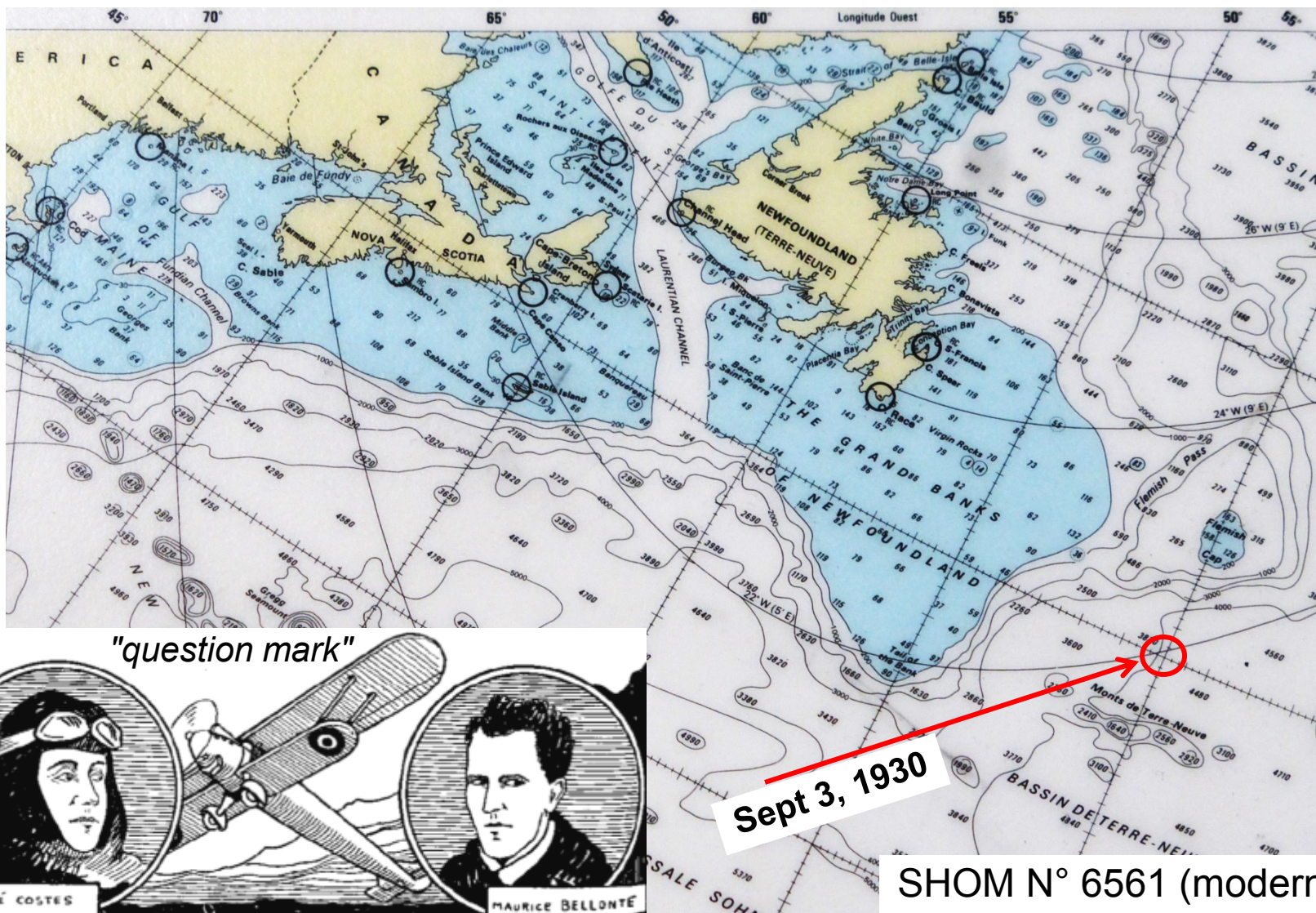


- « ? » to day, in Le Bourget Museum





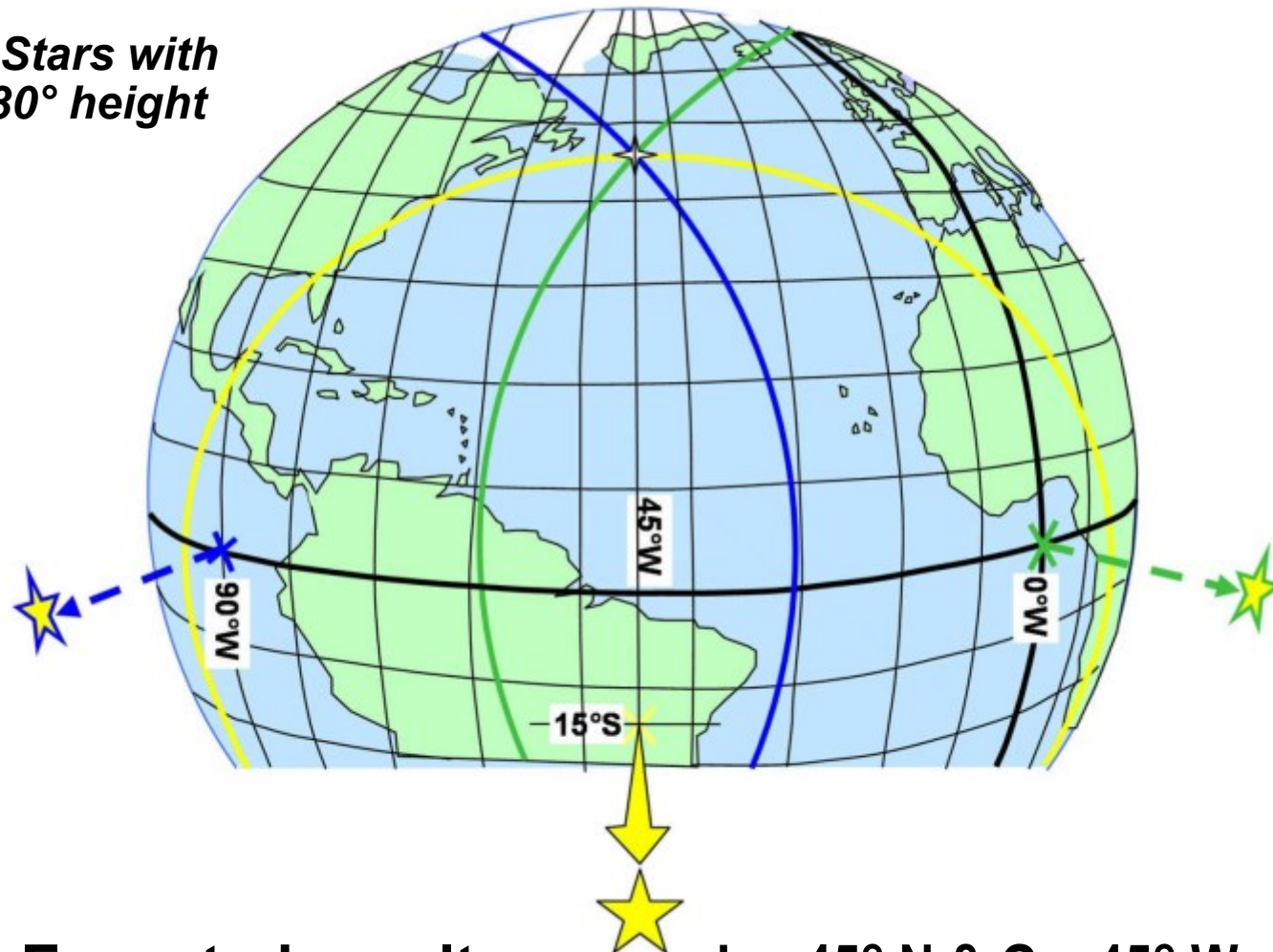
# Costes & Bellonte Paris-NY 1<sup>st</sup> crossing (1930)





# Costes & Bellonte canonic case with 3 Stars

*3 Stars with  
30° height*



◆ **Expected result:**

**$L = 45^\circ \text{ N}$  &  $G = 45^\circ \text{ W}$**



# Running ASTROLAB 3.1

*(now an old package)*

```
Programme de calcul du point - YRJ U5 du 12/07/11
***** METHODE DU PLAN DES SOMMETS *****
Licence individuelle IMO-12 cédée par Y.ROBIN-JOUAN

Nbre observations= 99
Continuer ? (O/N, Oui par défaut)

N° d'observation choisie dans fichier (0 pour arrêter) ? 4
2 / 9 / 30 à 5 h 0 mn 0 s
OBSERVATION N° 4 de AMER1
HAUTEUR CORRIGEE = 30.000 Deg déci
DECLINAISON = +0.000 Deg déci
ANGL HOR GW = +0.000 Deg déci
LONG ESTIM = 45° 0.0'W
LATI ESTIM = 45° 0.0'N

N° d'observation choisie dans fichier (0 pour arrêter) ? 5
2 / 9 / 30 à 5 h 0 mn 0 s
OBSERVATION N° 5 de AMER2
HAUTEUR CORRIGEE = 30.000 Deg déci
DECLINAISON = +0.000 Deg déci
ANGL HOR GW = +90.000 Deg déci
LONG ESTIM = 45° 0.0'W
LATI ESTIM = 45° 0.0'N

N° d'observation choisie dans fichier (0 pour arrêter) ? 6
2 / 9 / 30 à 5 h 0 mn 0 s
OBSERVATION N° 6 de AMER3
HAUTEUR CORRIGEE = 30.000 Deg déci
DECLINAISON = -15.000 Deg déci
ANGL HOR GW = +45.000 Deg déci
LONG ESTIM = 45° 0.0'W
LATI ESTIM = 45° 0.0'N

N° d'observation choisie dans fichier (0 pour arrêter) ? 0_
```

.....

```
TRANSPORT PAR CAP+DIST entre OBS ? (O/N, Non par défaut)

DEBUT CALCUL
CAS GENERAL
DOP= 5.57

PREDICTION
ALTI = -0.0 Naut
LATITUDE = 45 ° 0.0'N = +45.000 Deg déci
LONGITUDE = 45 ° 0.0'W = +45.000 Deg déci

RESULTATS
LATITUDE = 45 ° 0.0'N = +45.000 Deg déci
LONGITUDE = 45 ° 0.0'W = +45.000 Deg déci
ECART TYPE = +0.00 Naut

FIN CALCUL

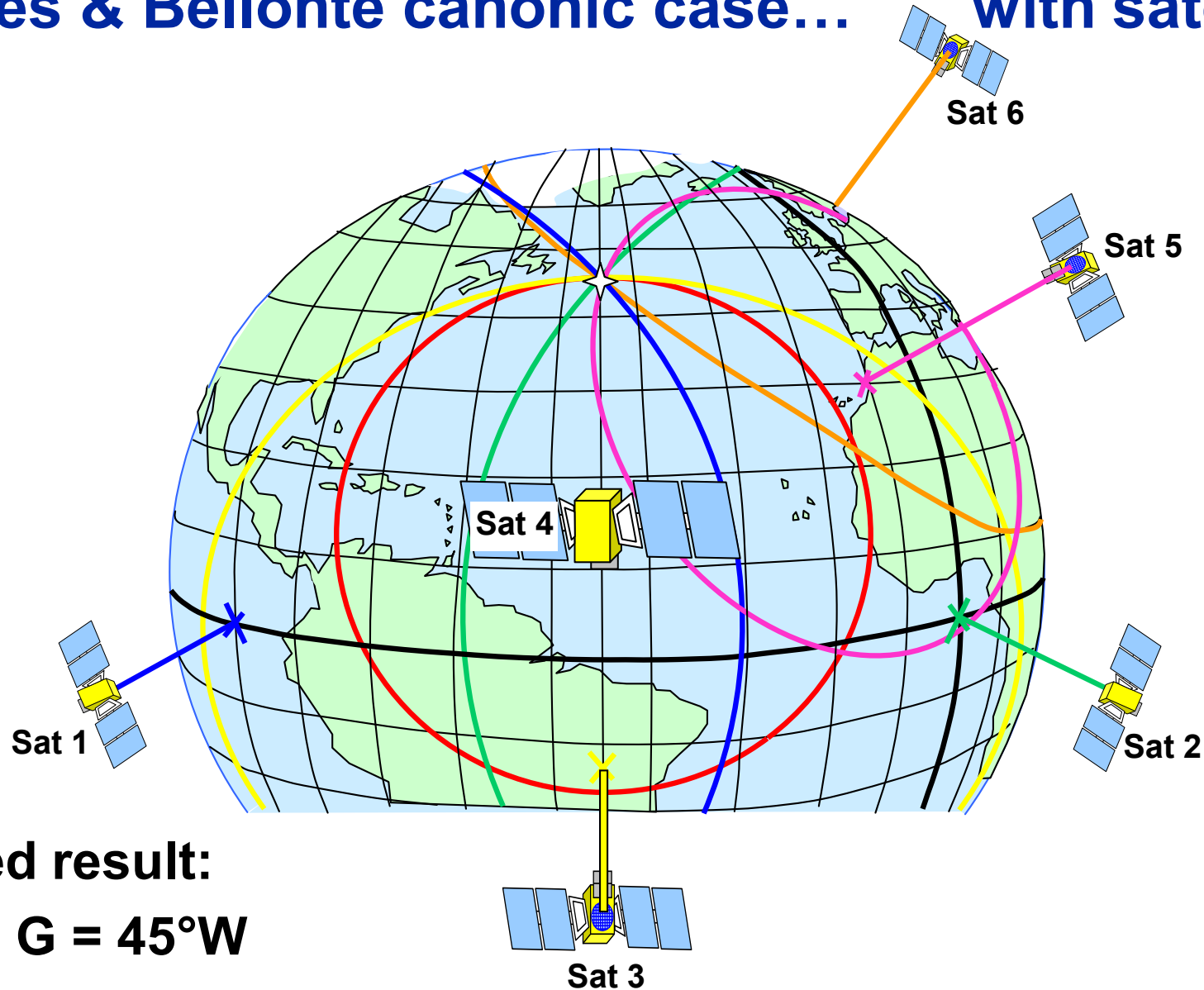
ECART POINT CALCULE A POINT ESTIME:
ECART = 0.00 Naut

ECRIURE POINT CALCULE DANS FICHIER ? (O/N, N par défaut)
```

*Result as expected, sharply*



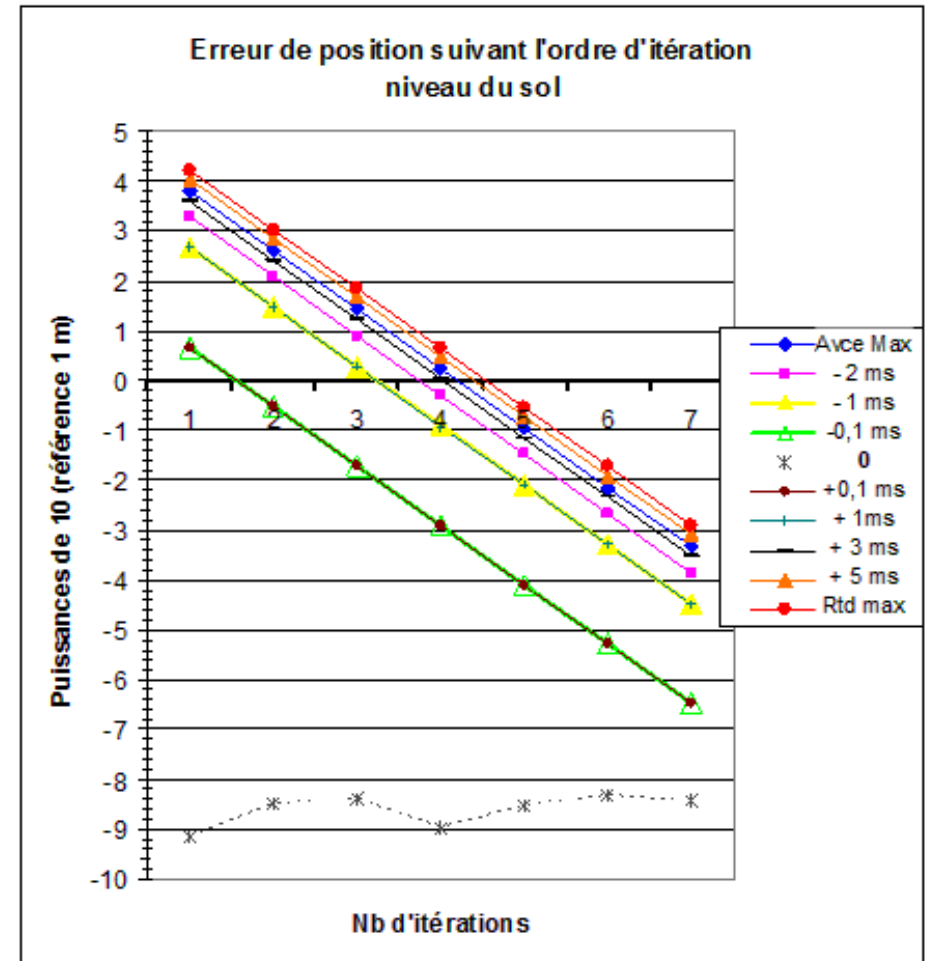
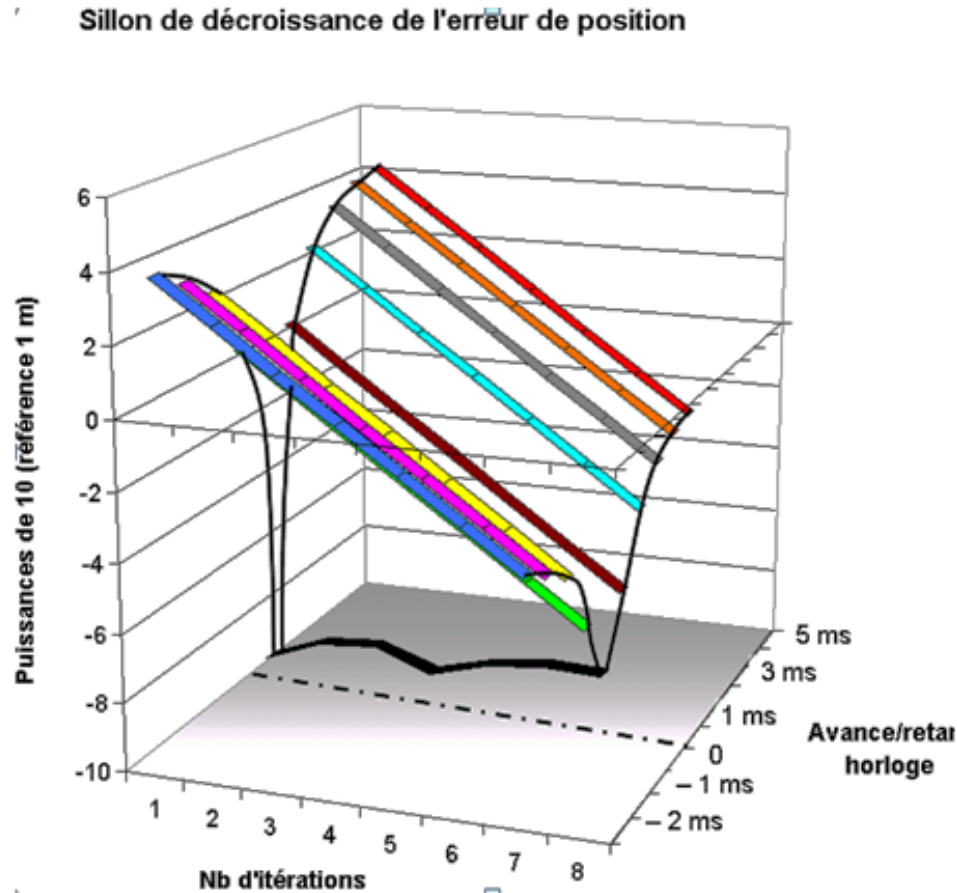
# Costes & Bellonte canonic case... with satellites



◆ Expected result:  
 $L = 45^\circ\text{N}$  &  $G = 45^\circ\text{W}$



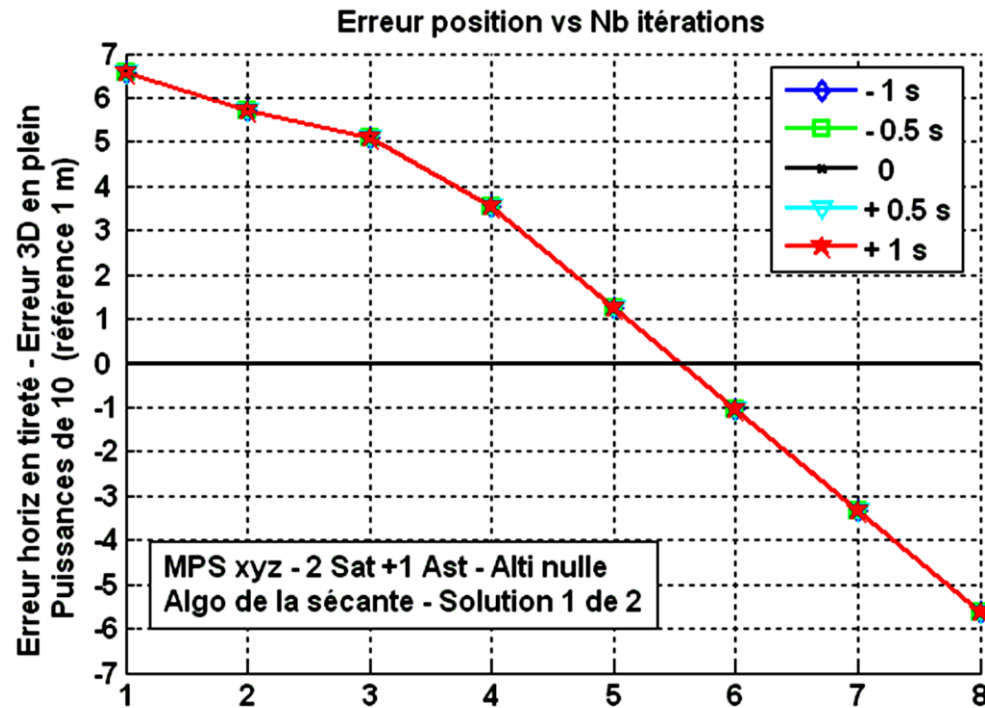
# Costes & Bellonte results with 4 GPS satellites (time shift $\tau \neq 0$ )



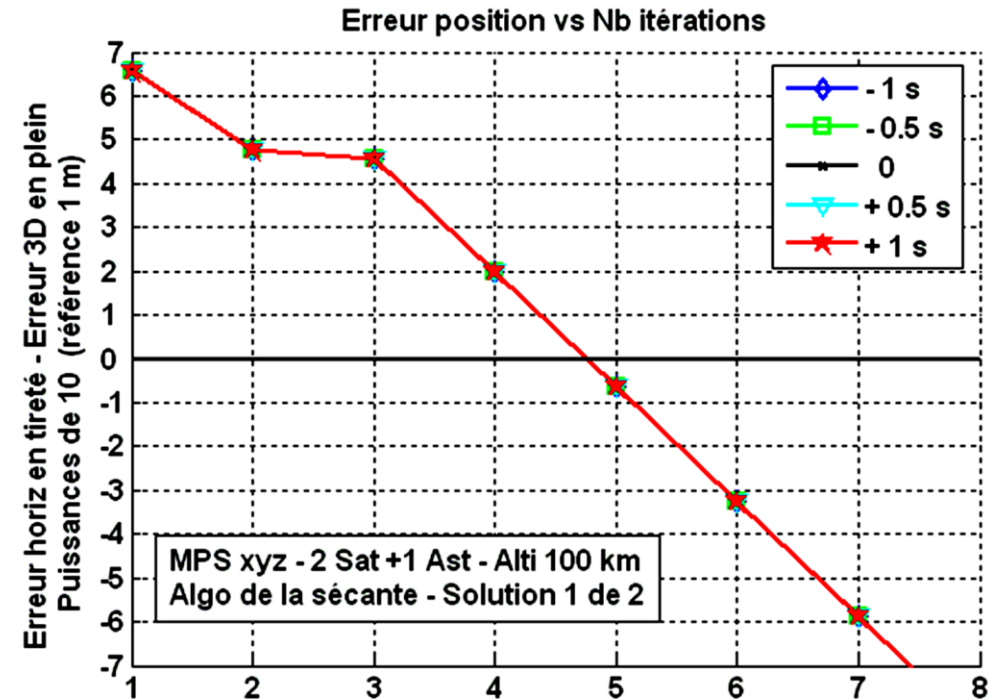
*Solution via MPS running Matlab (2006)*



# Costes & Bellonte results when hybridizing 2 Sat + 1 Star



Low altitude condition



High altitude condition

$\tau$  as a parameter in secondes (curves are superimposed)

***Solution via MPS running Matlab (2013)***



# A pragmatic case of landfall in Iroise sea (Brittany)

## ◆ Hybrid location based on 1 Star + a pair of Landmarks

- Venus :  $H_o = 10^\circ 53,8'$  at 18:00 on September 22, 2016 (*as an example*)
- Aperture angle sight between Créach & Armen lighthouses:  $\alpha = 84^\circ 44,9'$
- Computation of  $\zeta = 12,6'$
- ASTROLAB is run twice sequentially.

## ◆ Result : $48^\circ 15' \text{ N } 5^\circ 24,5' \text{ W}$ , 34 NM W from "Goulet de Brest"

## ◆ There is an alternative solution close to W Sein buoy

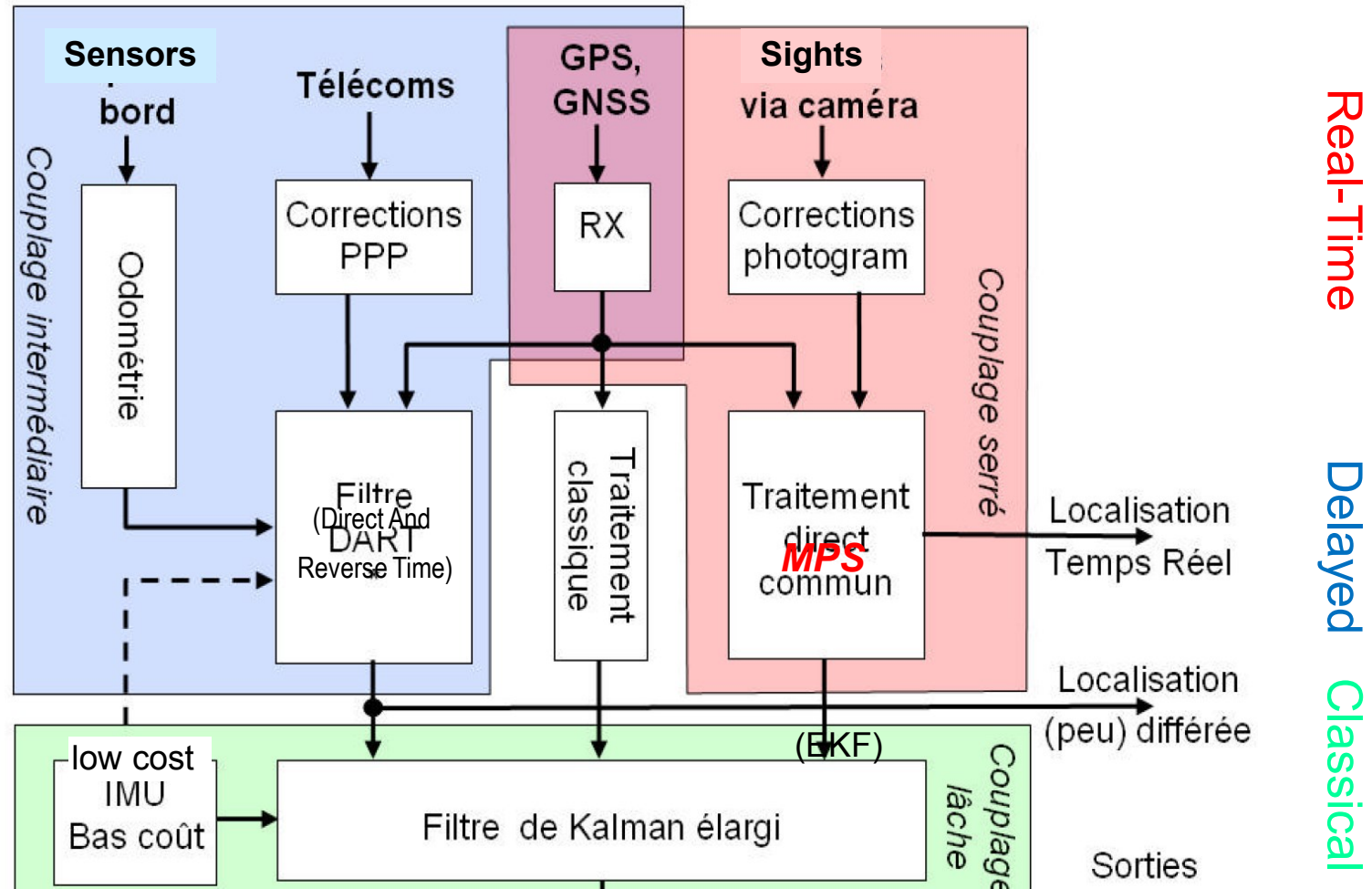








# A project for Autonomous Vehicles in ITS framework



**3 levels of coupling: tight + intermediate + loose (2016)**



## As a conclusion 1

- ◆ Exemple of an impact of Navigation history into modern life:  
Piri Reis Map (1513) on Turkish banknotes !



*The lost part of Piri Reis map has been sketched on the right side*



## As a conclusion 2

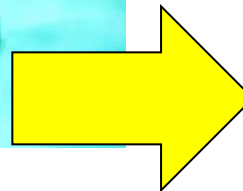
- ◆ From paper documents of F.Worsley & R.F.Scott, degraded by water...  
...to modern ruggedized tablets



On board J. Caird,  
TV movie, 2012



*"We had reached the  
naked soul of man"*  
*Sir E. Shackleton*  
1916



Fieldbook® CZ80,  
running Win or Android,  
as an example

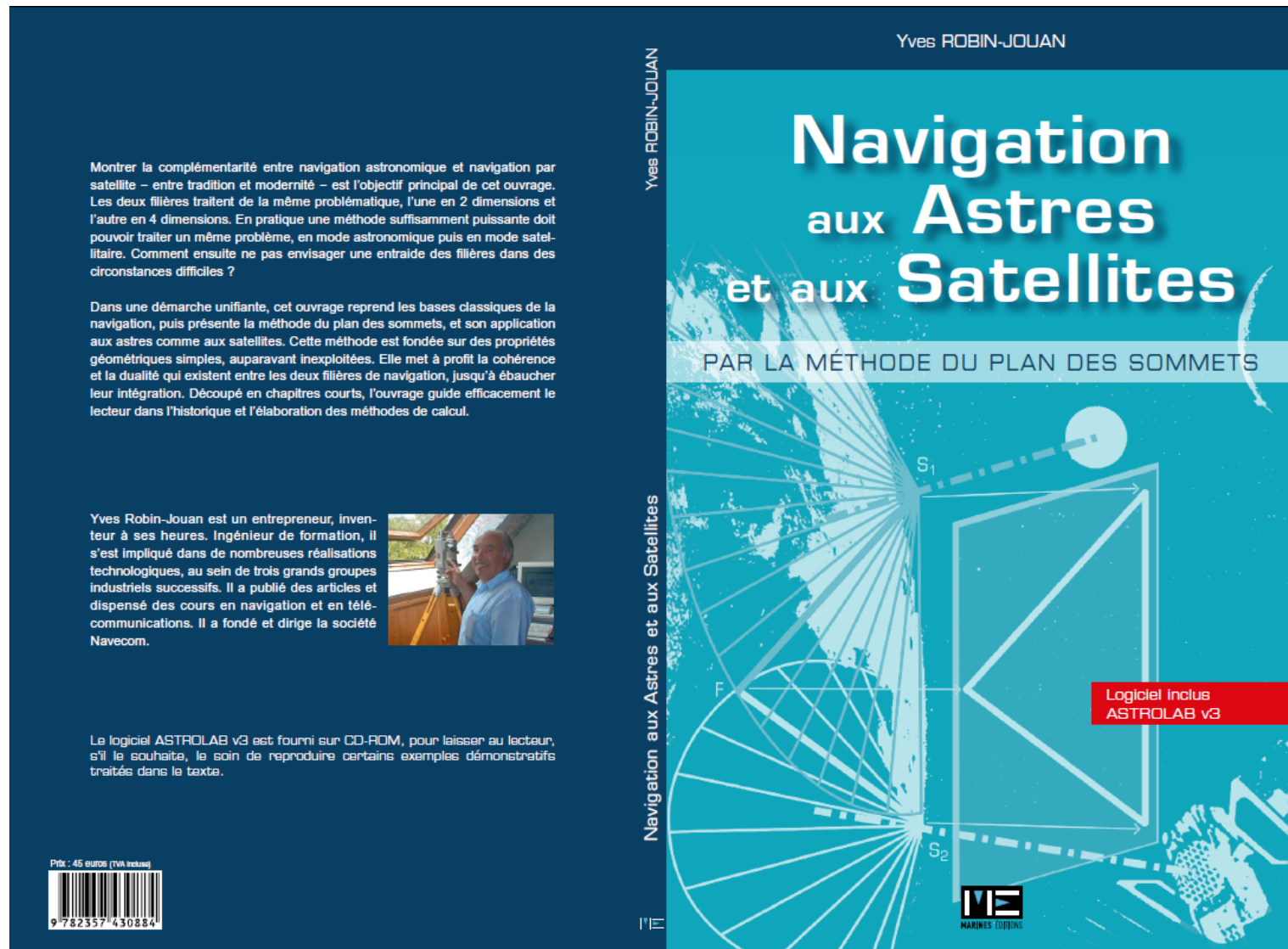


# Bibliography References

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- ◆ « **Nouvelle** Navigation Astronomique – Théorie & Pratique », **A. Yvon-Villarceau** et H. Aved de Magnac, 435 pages, Editions Gauthier-Villars, Paris 1877
- ◆ « Le problème de Douwes », Professeur G. Bodenez, 65 pages, Mémoire à l'Inspection de l'Enseignement Maritime, April 1976, officially registered in 1977
- ◆ "Sight reduction with matrices", R. Watkins & P.M. Janiczek, the Institute's professional forum, pages 447-448, Navigation ION, Winter 1978-1979
- ◆ « Le premier Paris - New York : Costes et Bellonte », 282 pages, M. Bellonte, Editions Plon, Paris 1976, ISBN 2-259-00101-7
- ◆ « L'avenir de la navigation astronomique dans la Marine Marchande », prix Daveluy 2013, Loïc Barbot, Professeur à l'Ecole Nationale Supérieure Maritime
- ◆ "Hybrid Navigation using celestial bodies and landmarks with the Method of Coplanar Vertices", Y. Robin-Jouan, NAVIGATION IFN, Vol 64 n°252, May 2017
- ◆ « Navigation aux Astres et aux Satellites par la Méthode du Plan des Sommets » Y. Robin-Jouan, Marines Editions, Jan 2013, ISBN 978-35743-088-4



# I recommend this good book !...





# ASTROLAB for maximum SW portability



**ASTROLAB v3.2 =  
MPS for Celestial Navigation  
and Coastal Navigation**