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Position from Observation of a Single Body

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ABSTRACT

A fundamental method of calculating the time difference between meridian transit and when a body is at its highest altitude allows direct correction for the effects of vessel velocity and rate of declination change. A novel graphical procedure for determining the time of highest altitude results in a simple way to obtain a fix from observations near meridian passage. Calculators or computers are not needed in the method. An example demonstrates the practical merit of the approach. Derivation of the equations is presented, and observational error sensitivity is discussed.

INTRODUCTION

To determine longitude from observations of a single body near meridian transit, the double altitude approach has been proposed, where the times for equal altitudes before and after meridian passage are averaged.[1] Several years ago, this author tried this method on trips to Catalina. From a known mid-channel position, longitudes were consistently calculated on the opposite side of the island! Analysis showed that boat speed was responsible—Reference 1 notes, "For a stationary observer . . . ," and "If there has been no change in declination . . . ." Further, on finding the time of meridian transit,[2] "This method is not reliable if there is a large northerly or southerly component of the vessel's motion, because the altitude at meridian transit changes slowly, particularly at low altitudes. At this time the change due to the vessel's motion may be considerably greater than that due to apparent motion of the body (rotation of the earth), so that the highest altitude occurs several minutes before or after meridian transit."

Figure 1 is a plot of altitude vs time, illustrating that north-south vessel motion produces a time of highest altitude different from that of a fixed observer. Declination rate of change has a similar effect on time of highest altitude, and may be dominant for the moon. East-west motion of the vessel has a much smaller effect, since vessel speed is significantly slower than the rate of change of Greenwich Hour Angle (GHA). Since the body is not on the observer's meridian at the time of highest altitude, and the latitude then is different from that at meridian passage, an error in altitude (Δh) results, but this is negligible for usual boat speeds and common latitudes.

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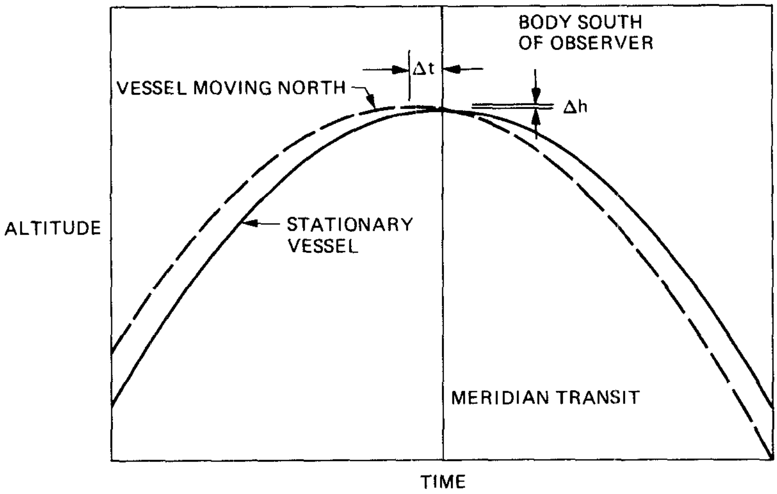

Fig. 1—Altitude vs Time

AN APPROACH

Thus, the time of highest altitude is different from the time of meridian transit. In using the double altitude method, this difference is independent of how long before meridian passage the initial altitude is measured, because it is due solely to vessel velocity and rate of declination change. A correction can be applied to the calculated time of meridian transit, or to the time average of equal altitudes. Presuming constant speed and course, a quite accurate calculation for the sun can be made by using the following equations (derivation of the equations and application to other bodies is explained in Appendix I):

$$\text{LAN} = \text{time of highest altitude} + \Delta t$$

$$\Delta t = \frac{48}{\pi}(\text{Sn} - d)(\tan \text{Lat} - \tan \text{Dec})$$

Where:
- LAN = Local Apparent Noon
- Δt = correction, seconds
- Sn = northward component of vessel motion, knots
- d = hourly change of declination, ′ (+ if Dec is changing northerly)
- Dec = declination (+ if north)
- Lat = latitude (+ if north)

The Catalina example at a dead reckoning (DR) Latitude of 34°N, Dec 10°S, $d = 0.9'$, Sn = −5.2 knots corresponds to a Δt of −1m19s. This translates into a longitude error of 20′W, or a 17 mile westerly error—quite close to that observed. Appendix II shows examples of Δts and Δhs for various conditions.

The equation for the calculation of the correction Δt has been plotted in Figure 2. Entering with Lat and Dec, paying attention to whether they are in the same hemisphere or the opposite one, a value of K can be selected. Read K only to the nearest whole unit; interpolation isn't warranted for usual small

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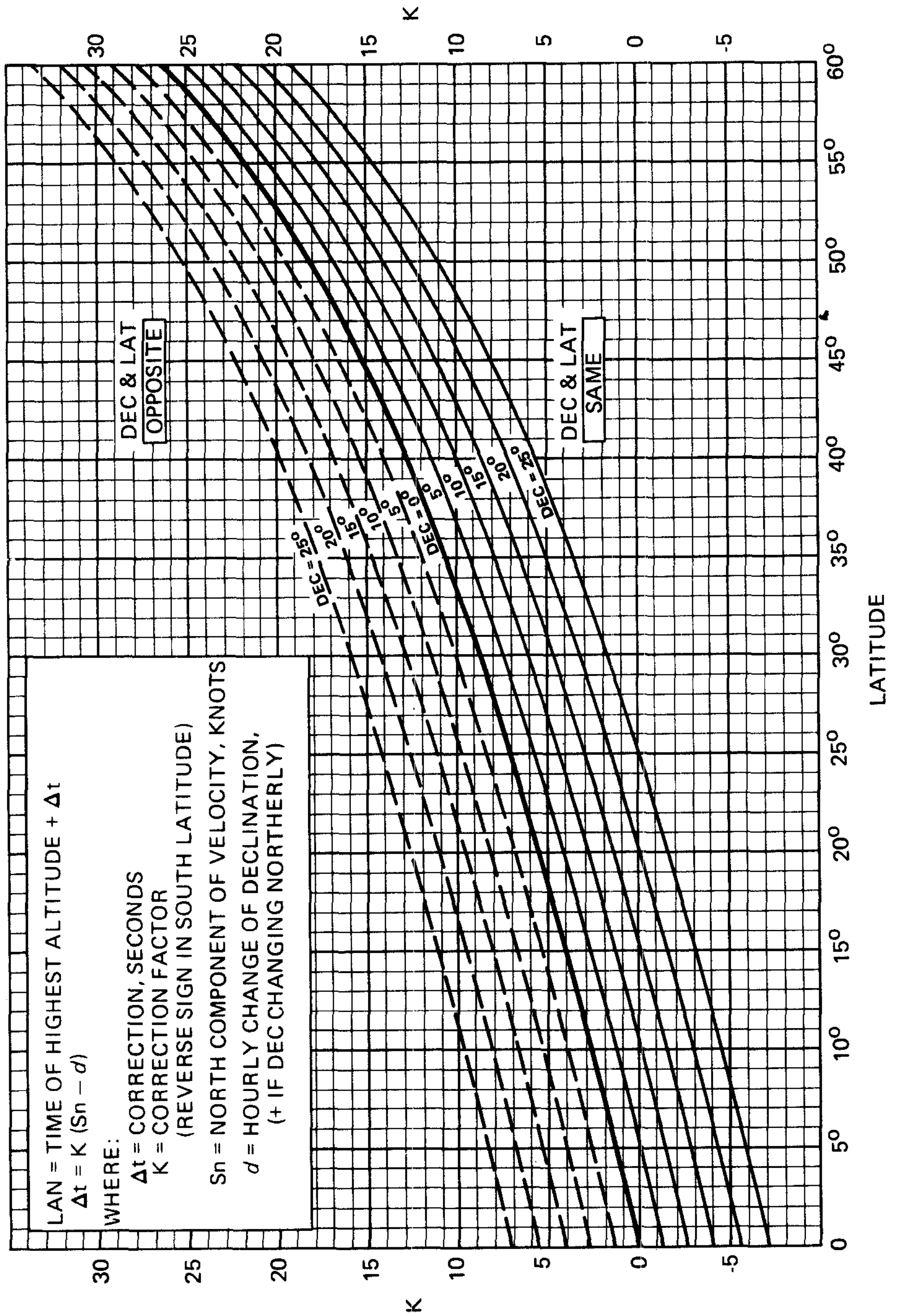

Fig. 2—Correction Factor for Time of Sun's Meridian Passage

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boat speeds. Multiply K by (Sn − *d*), keeping track of signs, and Δt results. Add it algebraically to the time of highest altitude to obtain LAN. Note that *d* may have a different sign from that used for declination calculation. For latitudes in the southern hemisphere, reverse the sign of K.

Figure 3 is a plot to aid in the calculation of Sn. Enter with S and true course (Cn), and read Sn. Note that Sn is negative for Cn between 90° and 270°.

ERROR SENSITIVITY

For normal sights, an observational error translates directly into a position error, so a 1′ error in sextant altitude (hs) results in a 1 mile error in the line of position. However, in attempting to determine longitude by double altitudes, any observer error is significantly magnified. Appendix III shows examples of Δt errors for a 1′ hs error ranging from 11 seconds to more than 3 minutes.

One way to minimize this error is to take the first sights 20 or 30 minutes before meridian transit, but longitude errors are still 2 to 10 times the hs error. Another way is to plot hs vs watch time (WT) and fair a straight line between the plotted points, to select the best hs. A further advantage of plotting hs is that the initial hs reading does not have to be exactly duplicated after meridian passage. A significant compensation for the work involved in plotting is that systematic errors or biases cancel.

THE METHOD

The following method was derived to obtain a reasonably accurate fix from a series of sights near meridian transit, presuming course and speed are constant.

1. About 20 or 30 minutes before meridian passage, record a run of five or

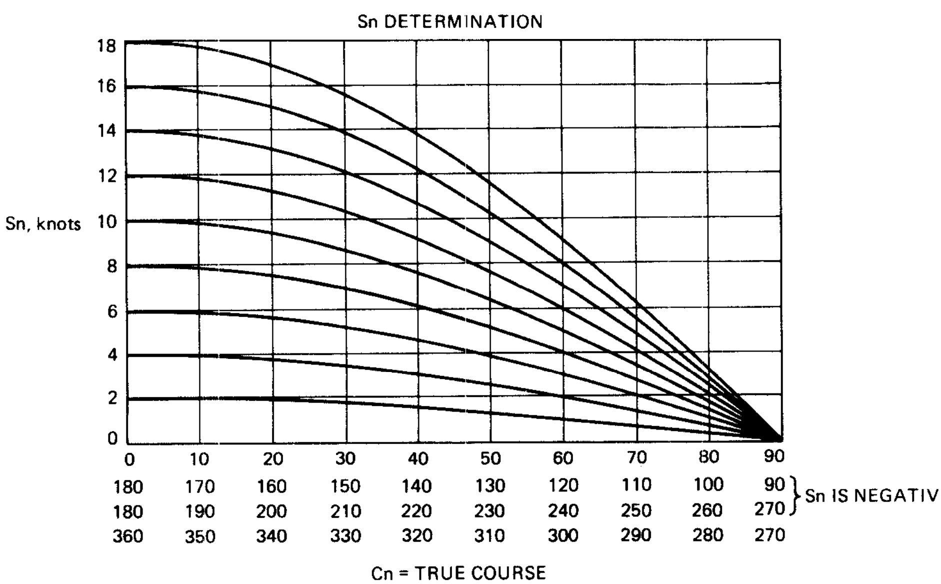

Fig. 3—Sn Determination

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more sights. Plot hs vs WT while waiting for the body to approach maximum altitude; Figure 4 shows such a plot.

2. Record enough sights near meridian transit to be sure that the body has attained and passed maximum altitude, and that meridian transit has occurred. Plot these on a compressed WT scale, so that the body's altitude arc can be faired through the normal hs scatter. This is used for latitude determination.
3. When hs has decreased to the highest value of the first run, record another run of at least five sights.

Now, a helpful trick—on the same plot, create a second WT scale directly under the first one. To facilitate averaging, start it an even number of minutes later than the first scale. Plot the last run of sights using this scale. (Use a different symbol for plotting this run, to avoid confusion with points from the first run.) Straight lines faired through each of the two sets of points will intersect at a pair of times when altitudes before and after meridian transit are equal. The average time from the two scales is the time of highest altitude. Another trick—average only the hours and minutes just before the time of the intersection, then read the seconds directly underneath it.

Then determine the Δt correction from the equation, or obtain K from Figure 2 and multiply by (Sn $-$ d), paying proper attention to signs. Apply this correction to the time of highest altitude to obtain the time of meridian transit, after which latitude and longitude can be calculated.

AN EXAMPLE

The following example illustrates the procedure for obtaining both latitude and longitude from a series of sun sights near LAN.

December 30, 1982

Known position at LAN	Lat	33°40.0′N
	Lo	118°16.6′W

LAN computation:

	Lo	118°16.6′W	
GHA sun at 1900	GMT	104°21.0′	
		13°55.6′	= 55m42s
	GMT	19-55-42	
	ZD	+8	(rev)
	ZT	11-55-42	LAN

The following data was recorded, Sun LL:

Course 210°	Speed 6.0 Knots	WE = 0-00
IC = +1.5′	Dip = −2.4′	

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WT	hs	WT	hs	WT	hs
11-28-21	32°34.0′	11-50-39	32°55.1′	12-19-54	32°42.6′
28-59	35.5′	51-41	56.1′	21-10	41.2′
30-00	34.0′	52-21	56.2′	22-51	37.6′
30-44	36.9′	53-31	58.5′	23-47	36.6′
31-24	38.5′	54-30	56.9′	24-50	35.0′
32-06	37.5′	55-04	57.5′	26-01	33.9′
32-42	40.3′	56-16	57.5′		
33-12	40.3′	56-52	57.6′		
		57-42	57.6′		
		58-42	59.9′		
		59-20	57.5′		
		12-00-58	55.1′		

After the first run of sights, plot hs vs WT. (Suggested scale: WT; ¼″ = 10sec, hs; ¼″ = 1′.) Start plot at 11-28-00; fair a straight line through the points (see Figure 4, line 1).

After the second run, plot hs, compressing the WT scale to ¼″ = 1 min. The hs scale may have to be broken (as in Figure 4) to get this run on the page. Fair a curve through these points (see curve 2).

After the third run of sights, establish the second WT scale, starting at 12-20-00 (an even number of minutes after 11-28). Plot the third run, again fairing a straight line through the points (see line 3). Average 11-31 and 12-23 to get 11-57, and read the time of equal hs below the intersection of the two lines as 11-57-09. This is the time of highest altitude.

Finally, perform the calculations:

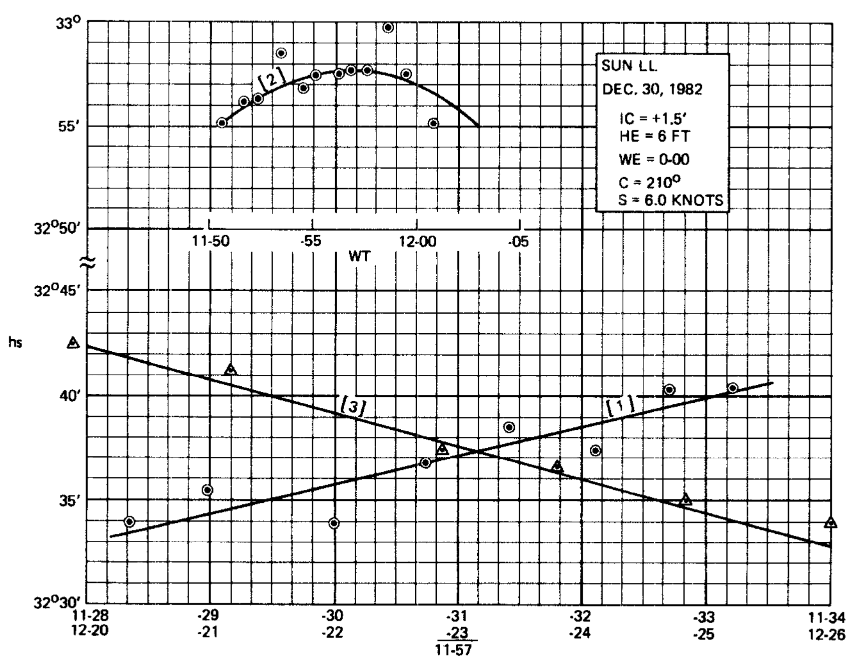

Fig. 4—Plot of Illustrative Example

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1. Determination of time correction

Lat ≅ 33°40′N
Dec ≅ 23°10′S (Lat and Dec OPPOSITE)

From Figure 2: K = 17. From Figure 3: for S = 6.0 and C_n = 210°

Sn = −5.2 knots

d = +0.2′ (Dec changing northerly)

Sn − *d* = −5.2 − 0.2 = −5.4′/hr

Correction = K (Sn − *d*) = 17 (−5.4) = −92s

= − 1m32s (−87 seconds from Δt equation)

2. Longitude determination

From Figure 4:

Time of highest altitude =	11-57-09	
WE =	0-00	
Correction =	(−) 1-32	
ZT =	11-55-37	
ZD =	+8	
GMT =	19-55-37	LAN
GHA sun at 1900 GMT =	104°21.0′	
55-37	13°54.3′	
Longitude =	118°15.3′W	
DR Longitude =	118°16.6′W	
Difference =	1.3′	

3. Latitude determination

Dec at 1900 GMT =		23°09.1′S
d = −0.2′	55m	−0.2′
Dec =		23°08.9′S

From Figure 4:

hs at LAN =	32°57.5′
IC =	+1.5′
Dip =	−2.4′
ha =	32°56.6′
Correction =	+14.8′
Ho =	33°11.4′
90 − Ho =	56°48.6′
Dec =	23°08.9′S
Lat =	33°39.7′N
DR Lat =	33°40.0′N
Difference =	0.3′

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DISCUSSION

The data for the example were taken while singlehanding a 31′ ketch on a clear day with a sharp horizon and a calm sea. The sun's altitude was low, and it's rate of change was slow. These observations were the first taken by the author in over two years, adding to the data scatter. Subsequent use of the method under significantly worse observation conditions has resulted in longitude errors of less than 5′.

The time correction equation was derived assuming constant heading and speed throughout the sight taking period. Maintaining constant heading is usually achievable, but where maintaining constant speed is not possible, using the average speed introduces only very small errors. For example, where Sn decreases linearly from 9.0 knots to 3.0 knots in 40 minutes, using the average speed of 6.0 knots introduces only a 2 second error in time of meridian transit.

While the practicality has been demonstrated, there are some drawbacks. The method requires that the sights be taken at a specific time, and this is not always convenient or possible. If overcast conditions threaten to restrict continual visibility of the body during the sight taking period, the first run of sights should be started earlier, to insure that they will encompass some portion of the last run. In extreme cases, the faired line from one of the runs can be extended as necessary to determine the time of highest altitude. Useful celestial bodies are probably limited to the sun and the moon, since twilight duration restricts the time before and after meridian passage when observations of stars and planets can be made. As noted in Appendix I, 7% should be added to the K value from Figure 2 for the moon.

To better predict the time of highest altitude, the time correction can be computed prior to taking sights. Sights should not be taken too rapidly, since several minutes should elapse during each run to enhance the ability to plot the altitudes. Note that the hs value used for latitude determination is that at the time of meridian transit, and not at the time of highest altitude. This results in a true fix, in that both latitude and longitude are determined at the same time, and also eliminates the small latitude error.

CONCLUSION

This example (and others) are evidence that the method is an effective way to obtain a fix from observations of the sun near meridian transit. Since plotting runs of sights is always recommended, no extra work is involved there, and the few computations are significantly less than reducing a pair of sights and plotting them to get a fix. (Some form of regression or least squares method could be used to determine the time of highest altitude,[3] but this places reliance on a calculator, thus disqualifying the method as a backup navigation system.)

Thus, if the satellite navigation receiver is out, the calculator is a carcass, and the sight reduction tables are moldy from disuse, this method will still allow the navigator to do his job. And for those who don't own any sophisticated equipment, it is simple enough to be used as a primary method.

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1. Bowditch, N., *American Practical Navigator,* Vol. 1, 1977 Edition, Defense Mapping Agency Hydrographic Center, Article 2114, "Double Altitudes," page 589.
2. Bowditch, N., *American Practical Navigator,* Vol. 2, 1981 Edition, Defense Mapping Agency Hydrographic Center, Article 733, "Finding Time of Meridian Transit," page 540.
3. Rogoff, M., *Calculator Navigation,* W. W. Norton & Co., 1979, pages 243–250.

APPENDIX I

DERIVATION OF THE EQUATIONS

Differentiating the basic altitude equation,

$$\sin \text{Hc} = \sin \text{Lat} \sin \text{Dec} + \cos \text{Lat} \cos \text{Dec} \cos \text{LHA}$$

yields,

$$\begin{aligned}\cos \text{Hc} \frac{d\text{Hc}}{dt} = {} & \sin \text{Lat} \cos \text{Dec} \frac{d\text{Dec}}{dt} \\ & + \cos \text{Lat} \sin \text{Dec} \frac{d\text{Lat}}{dt} \\ & - \cos \text{Lat} \cos \text{Dec} \sin \text{LHA} \frac{d\text{LHA}}{dt} \\ & - \cos \text{Lat} \sin \text{Dec} \cos \text{LHA} \frac{d\text{Dec}}{dt} \\ & - \sin \text{Lat} \cos \text{Dec} \cos \text{LHA} \frac{d\text{Lat}}{dt}\end{aligned}$$

For small LHA, $\cos \text{LHA} = 1$ and $\sin \text{LHA} = \pi \frac{\text{LHA}}{180}$

Substituting and combining:

$$\begin{aligned}\cos \text{Hc} \frac{d\text{Hc}}{dt} = {} & \left[\left(\frac{d\text{Dec}}{dt} - \frac{d\text{Lat}}{dt}\right)(\tan \text{Lat} - \tan \text{Dec}) \right. \\ & \left. - \pi \frac{\text{LHA}}{180} \frac{d\text{LHA}}{dt}\right] \cos \text{Lat} \cos \text{Dec}\end{aligned}$$

At maximum altitude, $\frac{d\text{Hc}}{dt} = 0$

So: $$\text{LHA} = \frac{180}{\pi}\left(\frac{d\text{Dec}}{dt} - \frac{d\text{Lat}}{dt}\right)\frac{(\tan \text{Lat} - \tan \text{Dec})}{d\text{LHA}/dt}$$

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If Meridian Transit = Time of highest altitude + Δt

Then: Δt = correction in seconds,

And: $$\frac{dLHA}{dt}\frac{\Delta t}{3600} = -\ LHA$$

So: $$\Delta t = \frac{648{,}000}{\pi}\left(\frac{dLat}{dt} - \frac{dDec}{dt}\right)\frac{(\tan Lat - \tan Dec)}{(dLHA/dt)^2}$$

Let: $$\frac{dLat}{dt} = \text{north component of velocity} = \frac{Sn}{60}$$

(Sn = north speed in Knots)

$$\frac{dDec}{dt} = \text{hourly change in declination} = \frac{d}{60}$$

(d in ′, + if Dec is changing northerly)

$$\frac{dLHA}{dt} = \text{hourly change in LHA} = \Delta LHA$$

ΔLHA = ΔGHA − ΔLo
ΔGHA = hourly change in GHA, degrees

ΔLo = hourly change in longitude, degrees
(+ if vessel is moving west)

Then: $$\Delta t = \frac{10{,}800}{\pi(\Delta LHA)^2}(Sn - d)(\tan Lat - \tan Dec)$$

Some examples for various bodies, assuming ΔLo = 0:

BODY	ΔGHA	$\frac{10{,}800}{(\Delta LHA)^2}$	Δ%	
Sun	15°	48		
Planets	15° + v	47.89	−0.3	(v = 1.0′)
Stars	15°02.5′	47.73	−0.6	
Moon	14°19′ + v	52.07	+8.5	(v = 5.1′)
		50.74	+5.7	(v = 16.4′)

Considering observational errors, the sun's value of K from Figure 2 can be used for all but the moon. For the moon, adding 7% to the sun's K value results in errors of only a few seconds.

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APPENDIX II

SOME EXAMPLES OF Δt AND Δh FOR VARIOUS CONDITIONS:

1. Sun

			Sn = 0		Sn = 6 knots		Sn = 20 knots	
Lat	Dec	*d*	Δt	Δh	Δt	Δh	Δt	Δh
34°N	23°S	0	0	0	1m41s	+5″	5m36s	+56″
34°N	0	−1.0′	10s	0	1m12s	+4″	3m36s	+38″
34°N	23°N	0	0	0	23s	+1″	1m16s	+12″
52°N	0	−1.0′	20s	0	2m17s	+8″	6m51s	+1′12″
52°N	23°S	0	0	0	2m36s	+8″	8m41s	+1′27″
82°N	7°N	−0.9′	1m36s	+1″	12m17s	+42″	37m13s	+6′19″

2. Moon

			Sn = 0	
Lat	Dec	*d*	Δt	Δh
34°N	7°S	−14.8′	3m13s	+24″

APPENDIX III

Δt ERRORS RESULTING FROM A 1′ ERROR IN hs

Lat 34°N	Sn = 0		
Time of first sight, minutes before meridian transit	Declination		
	23°S	0 *d* = 1.0′	23°N
5	3m20s	2m	46s
10	1m40s	1m	23s
15	1m07s	41s	15s
20	50s	31s	11s