



ϕ_1, ϕ_2 latitudes of start and finish
 B, D bearing and distance
 $\Delta\lambda$ longitude difference

Spherical trig gives

$$\begin{aligned} \cos(90^\circ - \phi_2) &= \cos(90^\circ - \phi_1) \cos D + \sin(90^\circ - \phi_1) \sin D \cos B \\ &= \sin \phi_1 \sin(90^\circ - D) + \cos \phi_1 \cos(90^\circ - D) \cos B \end{aligned}$$

Now, $\cos x = \frac{1 - \cos 2x}{2} \Rightarrow \cos x = 1 - 2 \cos^2 x$

Hence $\cos(90^\circ - \phi_2) = \sin \phi_1 \sin(90^\circ - D) + \cos \phi_1 \cos(90^\circ - D) (1 - 2 \cos^2 B) =$

$$= \underbrace{\sin \phi_1 \cos(90^\circ - D) + \cos \phi_1 \cos(90^\circ - D)}_{\cos(\phi_1 - (90^\circ - D))} - 2 \cos \phi_1 \cos(90^\circ - D) \cos B$$

$1 - 2 \cos^2(90^\circ - \phi_2)$

$$1 - 2 \cos^2(\phi_1 - (90^\circ - D))$$

$\Rightarrow \cos(90^\circ - \phi_2) = \underbrace{\cos(\phi_1 - (90^\circ - D))}_N + \underbrace{\cos \phi_1 \cos(90^\circ - D)}_{1-Q} \cos B$

$$\begin{aligned} \cos(x-y) + \cos(x+y) &= \frac{1 - \cos(x-y)}{2} + \frac{1 + \cos(x+y)}{2} = \\ &= 1 - \frac{\cos(x-y) - \cos(x+y)}{2} = 1 - \cos x \cos y \end{aligned}$$

$Q = 1 - \cos \phi_1 \cos(90^\circ - D) = \underbrace{\cos(\phi_1 - (90^\circ - D))}_N + \underbrace{\cos(\phi_1 + (90^\circ - D))}_P = N + P$

$\cos(90^\circ - \phi_2) = N + (1 - Q) \cos B$ with N, P, Q as above

In a similar manner, with

$$N = \cos(\phi_2 - \phi_1)$$

$$P = \cos(\phi_2 + \phi_1)$$

$$Q = N + P$$

you get

$$\cos \Delta\lambda = \frac{\cos D - N}{1 - Q}$$