Finding the intersection of two position circles by haversines only
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A stationary observer is assumed. East longitude positive. I have only derived the formulas for one geometry, there exist cases where these formulas have to be modified. Also, I give only one of the generally two intersections.

Call the Greenwich hour angles GHA_i i=1,2
Call the declinations δ_i
Call the true altitudes h_i

Calculate

$$F = \text{hav}(\delta_2 - \delta_1)$$

$$G = \text{hav}(\delta_2 + \delta_1)$$

$$\Delta t = GHA_1 - GHA_2$$

Calculate the distance D between the bodies:

$$\text{hav}\, D = F + [1 - (F + G)] \cdot \text{hav}\, \Delta t$$

Calculate the complement to the distance

$$D^* = 90° - D$$

and the complement to the first altitude

$$z_1 = 90° - h_1$$

Calculate

$$J = \text{hav}(h_2 - D^*)$$

$$K = \text{hav}(h_2 + D^*)$$

Now find angle α from

$$\text{hav}\, \alpha = \frac{\text{hav}\, z_1 - J}{1 - (K + J)}$$

Calculate

$$L = \text{hav}(\delta_2 - D^*)$$

$$P = \text{hav}(\delta_2 + D^*)$$

$$p_1 = 90° - \delta_1$$

Now find angle sum $\alpha+\beta$ from

$$\text{hav}(\alpha + \beta) = \frac{\text{hav}\, p_1 - L}{1 - (P + L)}$$

Calculate

$$\beta = (\alpha + \beta) - \alpha$$

$$R = \text{hav}(\delta_2 - h_2)$$

$$S = \text{hav}(\delta_2 + h_2)$$

Find complement to latitude from

$$\text{hav}\, \varphi^* = R + [1 - (S + R)] \cdot \text{hav}\, \beta$$

and latitude from

$$\varphi = 90° - \varphi^*$$

Calculate

$$U = \text{hav}(\delta_2 - \varphi)$$

$$V = \text{hav}(\delta_2 + \varphi)$$

$$z_2 = 90° - h_2$$

Find local hour angle from

$$\text{hav}\, t_2 = \frac{\text{hav}\, z_2 - U}{1 - (V + U)}$$

And, finally, longitude from

$$\lambda = t_2 - GHA_2$$

An example, using 4-figure natural haversines:

	body 1			body 2	
	°	'		°	'
GHA	318	12		42	6
δ	8	55		19	8
h	31	27		51	13
Δt	276	6			
F			0,0079		
G			0,0587		
hav D			0,425		
D	81	22			
D*	8	38			
z_1	58	33			
J			0,1319		
K			0,2488		
hav α			0,1731		
α	49	10			
L			0,0084		
P			0,0575		
p_1	81	5			
hav(α+β)			0,4433		
α+β	83	29			

β	34	19	
R			0,0764
S			0,3319
hav ϕ*			0,1279
φ*	41	55	
φ	**48**	**5**	
U			0,0625
V			0,3064
z_2	38	47	
hav t_2			0,0756
t_2	31	55	
λ	**-10**	**-** **11**	