<u>Navigation Formulae</u>

Sight Reduction (Intercept Method)

$H_c = \sin^{-1}(\sin(\text{Dec.}) \cdot \sin(\text{Lat.}) + \cos(\text{Dec.}) \cdot \cos(\text{LHA}) \cdot \cos(\text{Lat.}))$

$Z = \cos^{-1}((\sin(\text{Dec.}) \cdot \cos(\text{Lat.}) - \cos(\text{Dec.}) \cdot \cos(\text{LHA}) \cdot \sin(\text{Lat.})) / \cos(H_c))$

Z_n = If LHA < 180°, 360° - Z otherwise $Z_n = Z$

Sight Reduction (Sumner Line Method)

$t = \cos^{-1}(\sin(H_c) / \cos(\text{Dec.}) / \cos(\text{Lat.}) - \tan(\text{Dec.}) \cdot \tan(\text{Lat.}))$

In western longitudes, if the body is east, lon. = GHA + t otherwise lon. = GHA - t.

In eastern longitudes, subtract the above from 360°.

Time Sight

$t = \cos^{-1}((\sin(H_c) - \sin(\text{Dec.}) \cdot \sin(\text{Lat.})) / (\cos(\text{Dec.}) \cdot \cos(\text{Lat.})))$

(Note: This formula can also be used for the Sumner line method.)

Ex-Meridian

haversine(MZD) = haversine(TZD) - haversine(LHA) · cos(Dec.) · cos(Lat.)

…where:

- haversine = $(1 - \cos(\theta)) / 2$
- MZD = meridian zenith distance
- TZD = true zenith distance

Dip

$-0.0293° \cdot \sqrt{h}$

…where:

- h = height of eye in meters
- m = ft. · 0.3048

-or-

$-0.97' \cdot \sqrt{h}$

…where:

- h = height of eye in ft.

Refraction

$R = fR_0$

…where:

- $R_0 = -0.0167° / \tan(H_a + 7.32° / (H_a + 4.32°))$
- $f = 0.28 \cdot P / (T + 273)$
- P = pressure in mb
- mb = inHg · 33.86
- T = temperature in °C
- °C = (°F - 32) · (5/9)

Parallax in Altitude

$HP \cdot \cos(H_a)$

…where:

- HP = horizontal parallax
- $HP = \sin^{-1}$(radius of Earth / distance to body)
- radius of Earth = 6371 km or 4.25875×10^{-5} AU

Semi-diameter of the Moon

$0.2724 \cdot HP$

Lunars

$LD_c = \cos^{-1}(\sin(\text{Dec. }1) \cdot \sin(\text{Dec. }2) + \cos(\text{Dec. }1) \cdot \cos(\text{Dec. }2) \cdot \cos(\text{GHA }2 - \text{GHA }1))$

$LD_a = LD_s \pm IC \pm SD$*

$LD_o = LD_a + -dh_m \cdot A + -dh_s \cdot B + Q$

…where

- dh_m = refraction + parallax in altitude
- dh_s = refraction
- $A = (\sin(h_s) - \cos(LD) \cdot \sin(h_m)) / (\cos(h_m) \cdot \sin(LD))$
- $B = (\sin(h_m) - \cos(LD) \cdot \sin(h_s)) / (\cos(h_s) \cdot \sin(LD))$
- $Q = (0.5 \cdot (dh_m - dh_s)^2 \cdot \cot(LD) \cdot (1-A^2)) / 3438$

*Note: Include the augmentation of the Moon's SD = $0.3' \cdot \sin(\text{H Moon})$.

Great Circle Routes

Use the intercept sight reduction formulae. Substitute latitude of destination for declination and difference in longitude for LHA.

G.C. distance in NM = $(90° - H_c) \cdot 60$

Initial G.C. course angle = Z_n

Points Along the Great Circle Route

Substitute 90° - distance between points for declination and initial course angle for LHA.

Latitude of Point = H_c

Difference in Longitude of Point = Z

Rhumb Line Course Between Points

$\tan(C)$ = departure / difference in latitude

…where:

- departure = difference in longitude · cos(mid latitude)

Vertex of the Great Circle Route

$\cos(Lat_v) = \cos(Lat_1) \cdot \sin(C)$

$\sin(dLon) = \cos(C) / \sin(Lat_v)$

-or-

$\cos(dLon) = \tan(Lat_1) \cdot \cot(Lat_v)$

Composite Sailing

Use the dLon vertex formula to find longitude of limiting latitude point.

Dead Reckoning

$Lat_1 = Lat_f + t \cdot (V / 60) \cdot \cos(C)$

$Lon_1 = Lon_f + t \cdot (V / 60) \cdot \sin(C) / \cos(Lat_f)$

…where:

- t = time interval
- V = speed in kts.
- Lat_1, Lon_1 = DR position
- Lat_f, Lon_f = position of last fix

Distance Between Points

$60 \cdot \sqrt{((Lon_1 - Lon_f)^2 \cdot \cos^2(Lat_f) + (Lat_1 - Lat_f)^2)}$

Position of the Fix

$Lat_1 = Lat_f + (C \cdot D - B \cdot E) / G$

$Lon_1 = Lon_f + (A \cdot E - B \cdot D) / (G \cdot \cos(Lat_f))$

…where:

- $A = \cos^2(Z_1) + \cos^2(Z_2) + \ldots$
- $B = \cos(Z_1) \cdot \sin(Z_1) + \cos(Z_2) \cdot \sin(Z_2) + \ldots$
- $C = \sin^2(Z_1) + \sin^2(Z_2) + \ldots$
- $D = p_1 \cdot \cos(Z_1) + p_2 \cdot \cos(Z_2) \ldots$
- $E = p_1 \cdot \sin(Z_1) + p_2 \cdot \sin(Z_2) \ldots$
- $G = A \cdot C - B^2$
- p_n = intercept and Z_n = azimuth

Note: If the distance between the AP and fix exceeds 20 NM (see distance formula above), set $Lat_f = Lat_1$, $Lon_f = Lon_1$ and repeat the calculation (incl. sight reduction) until the distance between points is less than 20 NM.

Phase of the Moon

Haversine of the lunar distance [(1 - cos(LD)) / 2]

Amplitude

Celestial horizon:

sin(A) = sin(Dec.) / cos(Lat.)

*Sun LL ⅔ of diameter above visible horizon; Moon UL on visible horizon; stars & planets 1 Sun diameter above visible horizon

Visible Horizon:

sin(A) = (sin(Dec.) – sin(Lat.) · sin(H)) / (cos(Lat.) · cos(H))