Haversine Navigation

Abstract

This is to provide a practical means for cruise planning using either the computer spreadsheet, the hand held calculator, or applications (apps) available for the iPhone or iPad.

Background

Shortly after achieving the grade of Navigator with the Shrewsbury River Power Squadron in April of 1962, I took an active interest in the mathematics of sight reduction that went beyond the methods of HO 211 and HO 214. Preferring analytic over graphic voyage planning provided the motive for these studies. Please remember that this was done before the computer, GPS, and even the most basic hand held calculator. Today using data (waypoint information) from a Cruising Guide, it is easier and faster to plan with the computer.

The spherical trigonometry equations taught left an ambiguity when the initial course was close to due east or west. Further solutions using the Spherical Law of Cosines fall apart when the distances are very small; short distances are most important to the vast majority of weekend sailors. Haversines resolved these issues. Today there are dozens of websites that will give you the course and distance between two points on the earth given the Latitude and Longitude of each. Many of these sites report the distance in kilometers which require a definition of the radius of the earth. The haversine method herein described uses a unit radius; thus, a distance as an angle readily resolves to nautical miles; therefore, I am going to limit this to a simple definition of the haversine and practical topics for cruise planning with the spreadsheet on the computer and to solutions on the handheld calculator as well as on the iPhone and iPad.

The Haversine

The haversine is half a versed sine. The versed sine appeared in early trig tables but is no longer used. The versed sine or versine is 1 – cosine (Θ). Thus by definition

$$\text{Haversine}(\Theta) = \sin^2(\Theta/2) = (1 - \text{cosine}(\Theta)) / 2$$

Now we will apply this to our spherical trig application to determine the course and distance between two points.

Haversine Equations for Spherical Navigation

D = acos(1 – 2 * havD), where

havD = hav(CoLA - CoLB) + SinCoLA * SinCoLB * havDLo

C = acos(1 – 2 * havC), where

havC = (havCoLB - hav(CoLA - D)) / (sinCoLA * sinD)

Assumptions & Notes

The earth is a perfect sphere. This is accurate for the purpose of cruise planning within a reasonable area, e.g. Bahamas or Eastern Caribbean.

Neglecting the fact that the earth is an oblate spheroidal can result in errors of typically less than 0.5%; this is not significant for our purposes.

The spreadsheet and handheld calculator implementations shown here are valid only for the northwestern quadrasphere[1].

Haversine methodology will give errors when approaching antipodal points (on opposite sides of the earth); this is of minor consideration to the practical sailor.

Implementations

1. Spreadsheet

The Microsoft Excel workbook HaversineNavigation.xlsx consists of several worksheets:

- BB Navigation – This is the example using a portion of the Bowditch Bay chart featuring a fictitious April Fool's Day race both clockwise and counterclockwise around a set of four waypoints. (See Figures I & II)
- BBdata - This feed contains the waypoint data for the preceding worksheet. Items in the waypoint column are to be sorted alphabetically from A-Z; waypoint names must contain no spaces. (See Figure III)
- HaversinePlot – Worksheet to show that the haversine function is continuous over the range from 0^0 through 180^0. (See Figure IV)
- MI Navigation - This worksheet and the one below are for trip planning in and about the Merritt Island area.
- MIdata – Contains several user defined waypoints including those used recently for sailboat racing.

[1] a measure of land that is one fourth part of the world

- VI Navigation - This worksheet and the one below are for trip planning in and about the British Virgin Islands.
- VIdata – Contains many waypoints from several sources. Most are from **THE CRUISING GUIDE TO THE VIRGIN ISLANDS**, by Nancy & Simon Scott, 15th Edition • 2011-2012.
- HavNavHP 15C -
- Conversion - this worksheet and the following one were used for data conversion while planning a cruise in and about the British Virgin Islands.
- MapTechData

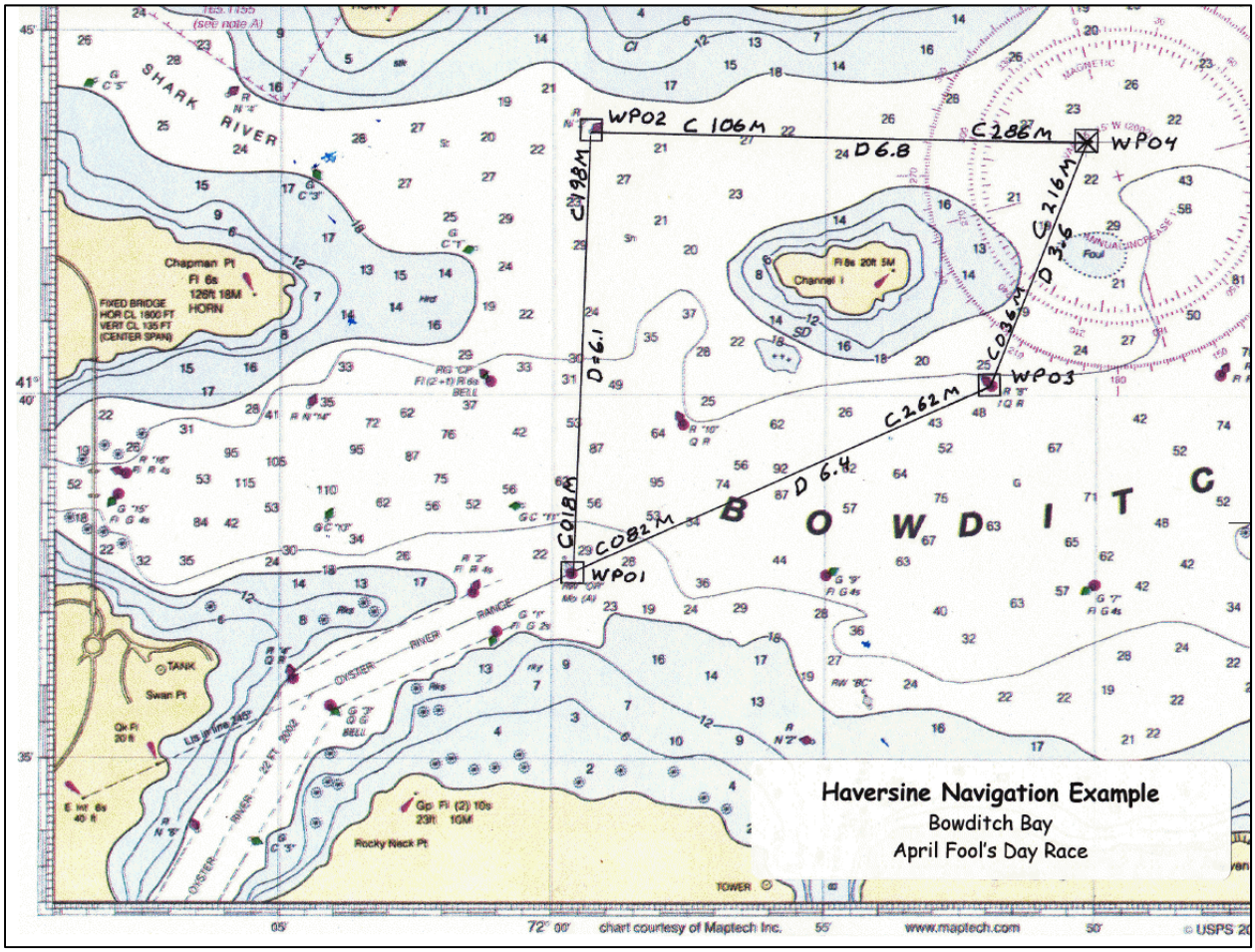


Figure I, Haversine Navigation Example, Bowditch Bay April Fool's Day Race

The magnetic courses for this fictitious race, both clockwise and counterclockwise were calculated using the workbook. Figure II shows the part of the worksheet BB Navigation that was used to create the counterclockwise courses. The distances between waypoints were also calculated in this worksheet.

The first step in cruise planning is to populate the data worksheet with your waypoints, their description, and their latitude and longitude. The navigation worksheet requires that latitude and longitude be supplied in format DD.dddd. You may choose to enter them directly into columns C and D in that format or use column C through H to aid you.

Once your data worksheet is populated you may now proceed to the navigation sheet where you enter your waypoints in rows one and two along with the magnetic variation in row 12. As you enter this data into the white spaces results will show immediately in the yellow shaded areas; the gray areas are for intermediate calculations. Once you have downloaded your HaversineNavigatiion.xlsx workbook you may explore it to see what is calculated in each cell and the formula used.

	A	B	C	D	E
1	From ->	WP01	WP03	WP04	WP02
2	To ->	WP03	WP04	WP02	WP01
3	From ->	Oyster River Entrance Buoy {RW "OR" Mo (A)}	Marker SE Channel Island {R "8" 1QR}	Waypoint NE of Channel Island	Marker East of Shark River {RN "2"}
4	To ->	Marker SE Channel Island {R "8" 1QR}	Waypoint NE of Channel Island	Marker East of Shark River {RN "2"}	Oyster River Entrance Buoy {RW "OR" Mo (A)}
5	Initial Course (True)	066	020	271	183
6	Initial Course (Magnetic)	082	036	286	198
7	Distance (nm)	6.4	3.6	6.8	6.1
8	Latitude A	41.6258	41.6683	41.7250	41.7267
9	Longitude A	71.9950	71.8650	71.8367	71.9883
10	Latitude B	41.6683	41.7250	41.7267	41.6258
11	Longitude B	71.8650	71.8367	71.9883	71.9950
12	Variation	15.25	15.25	15.25	15.25
13	LaA(rad)	0.7265	0.7272	0.7282	0.7283
14	LoA(rad)	1.2565	1.2543	1.2538	1.2564
15	LaB(rad)	0.7272	0.7282	0.7283	0.7265
16	LoB(rad)	1.2543	1.2538	1.2564	1.2565
17	CoLaA	0.8443	0.8435	0.8426	0.8425
18	CoLaB	0.8435	0.8426	0.8425	0.8443
19	dCoLa	0.0007	0.0010	0.0000	-0.0018
20	hav(CoLaA-CoLbB)	0.0000	0.0000	0.0000	0.0000
21	Dlo	0.0023	0.0005	-0.0026	-0.0001
22	hav(DLo)	0.0000	0.0000	0.0000	0.0000
23	hav(D)	0.0000	0.0000	0.0000	0.0000
24	D	0.0019	0.0011	0.0020	0.0018
25	hav(CoLaB)	0.1676	0.1672	0.1672	0.1679
26	CoLaA-D	0.8424	0.8425	0.8406	0.8408
27	hav(CoLaA-D)	0.1672	0.1672	0.1665	0.1666
28	hav(Co)	0.2992	0.0316	0.4922	0.9994
29	Co(rad)	1.1576	0.3571	1.5552	3.0922
30	Co(ddd.d)	66.3274	20.4631	89.1060	177.1707

Figure II, Part of Worksheet BB Navigation

	A	B	C	D	E	F	G	H
1	Waypoint	Description	Latitude	Longitude	Lat D	Lat MM.m	Lo D	Lo MM.m
2	WP01	Oyster River Entrance Buoy {RW "OR" Mo (A)}	41.6258	71.9950	41	37.6	71	59.7
3	WP02	Marker East of Shark River {RN "2"}	41.7267	71.9883	41	43.6	71	59.3
4	WP03	Marker SE Channel Island {R "8" 1QR}	41.6683	71.8650	41	40.1	71	51.9
5	WP04	Waypoint NE of Channel Island	41.7250	71.8367	41	43.5	71	50.2

Figure III, Worksheet BBdata

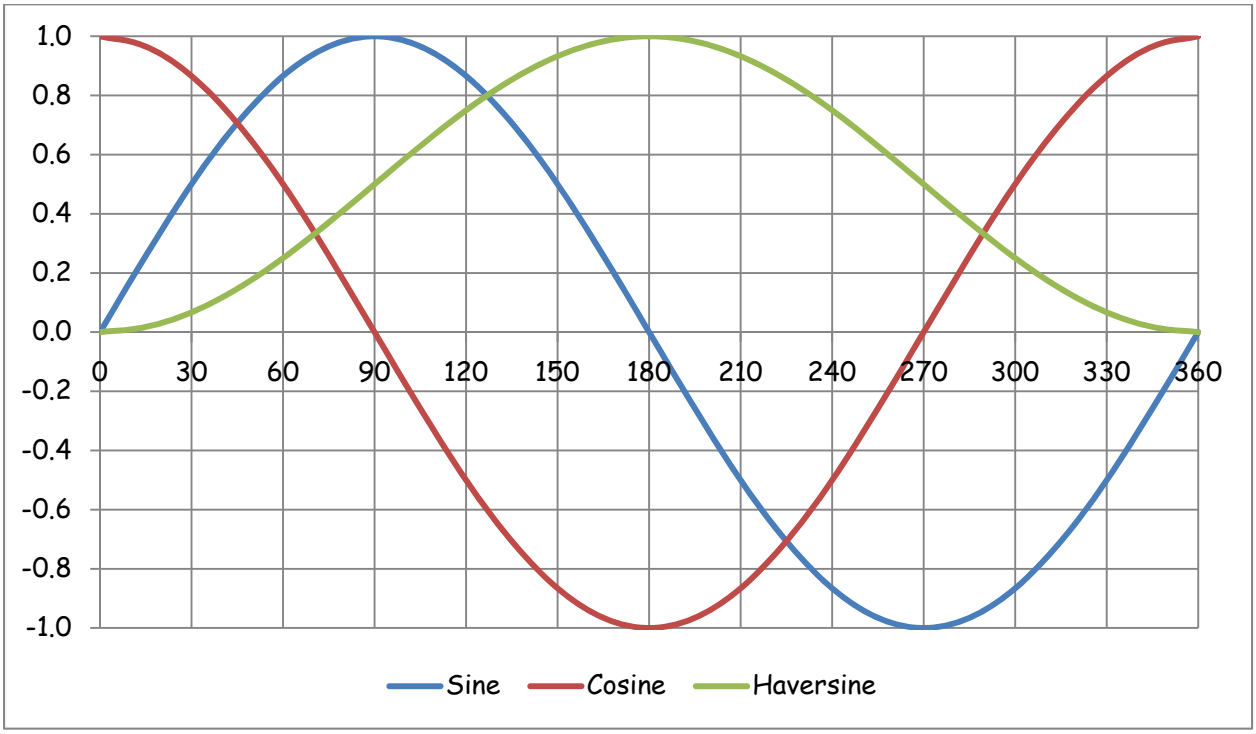


Figure IV, Plot of Sine, Cosine, & Haversine (0^0 to 360^0)

2. Calculator

The equations shown earlier are easily implemented into any handheld calculator with memory storage. I have chosen to show an application using the HP 15C calculator that uses RPN (Figure VI). This particular device has an app for use on the iPhone and iPad[2].

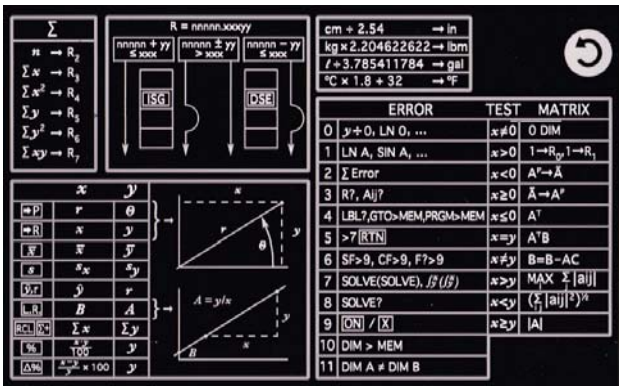
You only have to program your device once. The RPN programming steps are shown in Figure V[3].

[2] Interestingly it is easier to use on an iPad then it is on the actual calculator as the keypad is large.
[3] They may be more easily seen in worksheet HavNavHP 15C once you have downloaded the workbook HaversineNavigation.xlsx.

After programming, enter the latitude and longitude for the first waypoint into registers 5 and 6 in the DD.dddd format. Follow this with the latitude and longitude for the second waypoint into registers 8 & 9.

Enter the magnetic variation for the area into register 0. Press label A (f LBL A) to return the initial true course into register 2 and the distance nautical miles into register 3. They will also be in the X and Y registers respectively.

Front View



Rear View

Figure VI, HP 15C as seen on the iPad or iPhone

Press label B (f LBL B) to return the initial magnetic course into register 1. It will also be in the X register at this time.

Haversine navigation with the calculator is more cumbersome and less fast then the Excel workbook method; however, when it is available on your cell phone it makes up for these deficiencies with its convenience.

HP 15C Calculator

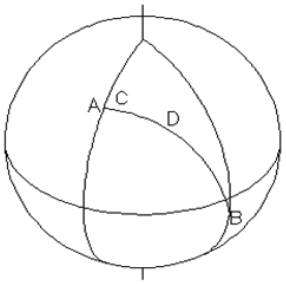
Haversine Navigation

Step	Action	Code	
000			
001	LBLA	42,21,11	A) Start
002	RCL5	45 5	
003	GSB2	32 2	Complement Angle
004	RCL8	45 8	
005	GSB2	32 2	Complement Angle
006	-	30	DiffCoLat
007	GSB0	32 0	Haversine Dlo
008	RCL5	45 5	
009	GSB2	32 2	Complement Angle
010	SIN	23	
011	RCL8	45 8	
012	GSB2	32 2	Complement Angle
013	SIN	23	
014	*	20	
015	RCL6	45 6	
016	RCL9	45 9	
017	-	30	Dlo
018	CF0	43,5,0	Clear Flag 0
019	X<0?	43,30,1	if(Dlo>0
020	SF0	43,4,0	then Set Flag 0
021	GSB0	32 0	
022	*	20	
023	+	40	
024	GSB1	32 1	ArcHaversine
025	STO3	44 3	
026	RCL8	45 8	
027	GSB2	32 2	Complement Angle
028	GSB0	32 0	Haversine
029	RCL5	45 5	
030	GSB2	32 2	Complement Angle
031	RCL3	45 3	
032	-	30	
033	GSB0	32 0	Haversine
034	-	30	
035	RCL5	45 5	
036	GSB2	32 2	Complement Angle
037	SIN	23	
038	RCL3	45 3	
039	SIN	23	

Step	Action	Code	
040	*	20	
041	/	10	
042	GSB1	32 1	ArcHaversine
043	IFF0	43,6,0	
044	GTO3	22 3	
045	3	3	subtracts 360
046	6	6	
047	0	0	
048	XY	34	
049	-	30	
050	LBL3	42,21,3	3) Jump location
051	STO2	44 2	Initial Course T
052	6	6	
053	0	0	
054	STO*3	44,20,3	Distance NM
055	XY	34	Results
056	RCL3	45 3	
057	XY	34	
058	RTN	43 32	
059	LBL0	42,21,0	0) Haversine
060	COS	24	HavΘ=(1-cosΘ)/2
061	1	1	
062	XY	34	
063	-	30	
064	2	2	
065	/	10	
066	RTN	43 32	
067	LBL1	42,21,1	1) ArcHaversine
068	2	2	Θ=acos(1-2*HavΘ)
069	*	20	
070	1	1	
071	XY	34	
072	-	30	
073	ACOS	43 24	
074	RTN	43 32	
075	LBL2	42,21,2	2) Compliment of
076	SIN	23	angle
077	ACOS	43 24	
078	RTN	43 32	

Step	Action	Code	
079	LBL B	42,21,12	B) Init Co M
080	RCL 2	45 2	Init Co T
081	RCL 0	45 0	Variation
082	+	+	
083	STO 1	44 1	
084	3	3	
085	6	6	
086	0	0	
087	X>Y?	43,30,7	
088	GTO 4	22 4	
089	-	10	
090	STO 1	44 1	
091	LBL 4	42,21,4	
092	RCL 1	45 1	Initial Course M
093	RTN	43 32	
094			
095			
096			
097			
098			
099			

7	8 LaB	9 LoB
4	5 LaA	6 LoA
1 Init Course M	2 Init Course T	3 Distance NM
0 Variation		



Haversine Equations for Spherical Navigation:

havD=hav(CoLA-CoLB)+SinCoLA*SinCoLB*havDLo

havC=(havCoLB-hav(CoLA-D))/(sinCoLA*sinD)

Use:

Enter LatA, LongA, LatB, & LongB into registers 5, 6, 7, & 9 respectively in DD.dddd format. Enter Magnetic Variation for the area into register 0.

Press Label A to return Initial True Course into register 2 and Distance in Nautical Miles in register 3. They will also be in x & y respectively.

Press Label B to reture Initial Magnetic Course into register 1. It will also be in the x register.

HaversineNavigation.xlsx / HavNavHP15C

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Figure V, View of the HavNav worksheet with programming steps and instructions[4]

Definitions/Abbreviations/Acronyms

ahav	ArcHaversine (angle whose haversine is.. .)
Cos	Cosine
DLo	Difference in Longitude
Hav	Haversine
LA	Latitude point A
LB	Latitude point B
LoA	Longitude point A
LoB	Longitude point B
RPN	Reverse Polish Notation
Sin	Sine

[4] Complementing an angle takes less programming steps with the Sin aCos method than subtracting the angle from 90.

Websites of interest:

- Haversine Navigation.xlsx
- Wikipedia on Haversine Formula
  - The haversine formula
  - The law of haversines
- HP 15C Scientific Calculator Resurrected[5]
- HP 15C Owner's Handbook
- Calculate distance, bearing and more between Latitude/Longitude points[6]

References

Bowditch (1958 edition)

- Index for the Haversine:
  - defined p929 -p1032
  - table 34 p1421-p1456[7]
  - explained p1197p
- Table 34 contains 56 pages 5 degrees per page of LogHav & NatHav
- Paragraph 822 p232
- Paragraph 2109 p528

[5] Reports of the demise of the HP 15C are highly exaggerated; HP has brought it back as the result of customer demand

[6] Cumbersome to use but with good reading below the calculations

[7] Printed tables of haversines, arc haversines, and their logarithms aided navigators in the days prior to the modern digital calculator and computer. Fifty-six pages of Bowditch have been replaced by the iPhone.