Finding the intersections of two position circles by logarithmic calculation
Lars Bergman, 26 February 2015

A stationary observer is assumed. Spherical earth. East longitude positive. I have only derived the formulas for one geometry, there may exist cases where these formulas have to be modified.

Call the Greenwich hour angles	GHA_i	i=1,2
Call the declinations	δ_i	
Call the true altitudes	h_i	

Calculate the distance D between the bodies:

$\cos D = \sin\delta_1 \sec\psi \sin(\psi + \delta_2)$, where $\tan\psi = \cot\delta_1 \cos|GHA_1 - GHA_2|$

Calculate the angles α and β :

$\text{hav}\,\alpha = \sec h_1 \csc D \cos s_\alpha \sin(s_\alpha - h_2)$, where $s_\alpha = \frac{1}{2}(h_1 + h_2 + D)$

$\text{hav}\,\beta = \sec\delta_1 \csc D \cos s_\beta \sin(s_\beta - \delta_2)$, where $s_\beta = \frac{1}{2}(\delta_1 + \delta_2 + D)$

Now, calculate the latitude φ :

$\sin\varphi_k = \sin\delta_1 \sec\gamma_k \sin(\gamma_k + h_1)$, where $\tan\gamma_k = \cot\delta_1 \cos|\alpha \pm \beta|$, with k=1,2 depending on + or - .

For longitude λ use the common time sight formula:

$\text{hav}(GHA_1 + \lambda_k) = \sec\varphi_k \csc p_1 \cos s_k \sin(s_k - h_1)$, where $p_1 = 90° - \delta_1$ and $s_k = \frac{1}{2}(\varphi_k + p_1 + h_1)$

Note that $\csc p_1 = \sec\delta_1$ so this logarithm is already available.

An example, the same as in Robin Stuart’s paper, but with tenth of minutes of arc only:

δ_1	-7°51,5′	log cot 0,86006 n	log sin 9,13585 n
GHA_1	101°29,5′		
GHA_2	147°17,9′		
	45°48,4′	log cos 9,84328	
ψ	-78°48,0′	log tan 0,70334 n	log sec 0,71167
δ_2	-7°48,6′		
	-86°36,6′		log sin 9,99924 n
D	45°21,4′	log csc 0,14783	log cos 9,84676
h_1	28° 2,5′	log sec 0,05420	
h_2	33°25,7′		
	106°49,6′		
s_α	53°24,8′	log cos 9,77527	
h_2	33°25,7′		
	19°59,1′	log sin 9,53374	
α	69°26,1′	log hav 9,51104	

D	45°21,4′	log csc 0,14783	
δ_1	-7°51,5′	log sec 0,00410	
δ_2	-7°48,6′		
	29°41,3′		
s_β	14°50,6′	log cos 9,98526	
δ_2	-7°48,6′		
	22°39,2′	log sin 9,58563	
β	93°14,2′	log hav 9,72282	
α	69°26,1′		
$\|\alpha-\beta\|$	23°48,1′	log cos 9,96140	
δ_1	-7°51,5′	log cot 0,86006 n	log sin 9,13585 n
γ	-81°25,3′	log tan 0,82146 n	log sec 0,82634
h_1	28° 2,5′		
	-53°22,8′		log sin 9,90450 n
$\boldsymbol{\varphi}$	**47°21,9′**	log sec 0,16949	log sin 9,86669
p_1	97°51,5′	log csc 0,00410	
h_1	28° 2,5′		
	173°15,9′		
s	86°38,0′	log cos 8,76883	
h_1	28° 2,5′		
	58°35,5′	log sin 9,93119	
LHA	-31°44,0′	log hav 8,87361	
GHA_1	101°29,5′		
$\boldsymbol{\lambda}$	**-133°13,5′**		

The other solution is:

$\|\alpha+\beta\|$	162°40,3′	log cos 9,97983 n	
δ_1	-7°51,5′	log cot 0,86006 n	log sin 9,13585 n
γ	81°46,4′	log tan 0,83989	log sec 0,84439
h_1	28° 2,5′		
	109°48,9′		log sin 9,97349
$\boldsymbol{\varphi}$	**-64° 1,2′**	log sec 0,35847	log sin 9,95374 n
p_1	97°51,5′	log csc 0,00410	
h_1	28° 2,5′		
	61°52,8′		
s	30°56,4′	log cos 9,93334	
h_1	28° 2,5′		
	2°53,9′	log sin 8,70384	
LHA	-36°51,5′	log hav 8,99975	
GHA_1	101°29,5′		
$\boldsymbol{\lambda}$	**-138°21,0′**		