Z_1 is that of the zenith of a place whose latitude is the same as that of the observer, and over whose meridian the body X is passing. P is the projection of the celestial pole and the circle is that of the equinoctial.

If the arc Z_1X can be found, the latitude of the place whose zenith is at Z_1 and, therefore, the observer's latitude, can also be found.

If l, d, z and h denote the observer's latitude, the body's declination, the body's zenith distance, and the time of the meridian passage of the body, respectively, we have, from the triangle PZX:

$$\cos h = \frac{\cos z - \sin l \sin d}{\cos l \cos d}$$

when l and d have the same name, and:

$$\cos h = \frac{\cos z + \sin l \sin d}{\cos l \cos d}$$

when l and d have different names.

When l and d have the same name:

$$\cos z - \sin l \sin d = \cos l \cos d \cos h$$

i.e.

$$\cos z - \sin l \sin d = \cos l \cos d(1 - \text{vers } h)$$

i.e.

$$\cos z - \sin l \sin d = \cos l \cos d - \cos l \cos d \text{ vers } h$$

i.e.

$$\cos z + \cos l \cos d \text{ vers } h = \cos l \cos d + \sin l \sin d$$

i.e.

$$\cos z + \cos l \cos d \text{ vers } h = \cos (l \sim d)$$

i.e.

$$\cos z + \cos l \cos d \text{ vers } h = 1 - \text{vers } (l \sim d)$$

i.e.

$$\text{vers } (l \sim d) = 1 - \cos z - \cos l \cos d \text{ vers } h$$

i.e.

$$\text{vers } (l \sim d) = \text{vers } z - \cos l \cos d \text{ vers } h$$

Similarly, when l and d have different names:

$$\text{vers } (l + d) = \text{vers } z - \cos l \cos d \text{ vers } h$$

In general:

$$\text{vers}\,(l \pm d) = \text{vers}\,z - \cos l \cos d\,\text{vers}\,h \quad . \quad . \quad (1)$$

Now $(l \pm d)$ is the meridian zenith distance of X at the place whose zenith is at Z_1. Let this be denoted by z_1. The latitude of this place and, therefore, the observer's latitude, may thus be found:

$$\text{Latitude of observer} = (l \pm d) \pm d$$

that is

$$l = z_1 \pm d$$

The above treatment is a modified form of that first given in 1754 by Cornelius Douwes, a figure famous in the history of the double-altitude problem. Douwes's investigation was modified by the Rev. James Inman D.D., whose famous nautical tables were first published in 1821. Inman's modification, used above, consists in adapting the formula to the tables of natural versines and haversines.

The ex-meridian method described above requires the use of a latitude by account, which should approximate to the observer's actual but unknown latitude. If the latitude found differs materially from that used it is necessary to repeat the computation, this time using the calculated latitude in place of the one initially used. Moreover, it is necessary for the observer to know his longitude; knowledge of this being required to find h, which figures in the computation.

It may readily be shown that:

> Error in z_1 (and therefore error in calculated latitude) is proportional to cos latitude sin azimuth × error in h.

It follows, therefore, that the smaller the latitude or the nearer the azimuth to 90°, the greater will be the error in latitude consequent upon an error in h. Knowledge of correct time is all important in the ex-meridian problem.

It was early realized that when using stars for finding latitude in accordance with the ex-meridian method, those with big declinations gave the best results, this because of their relatively slow rates of change of altitude. The Pole Star, therefore, is admirably suited for the purpose; and accurate Pole Star tables have