Chapter 27

Equation of Time

Due to the eccentricity of its orbit, and to a much less degree due to the perturbations by the Moon and the planets, the Earth's heliocentric longitude does not vary uniformly. It follows that the Sun appears to describe the ecliptic at a non-uniform rate. Due to this, and also to the fact that the Sun is moving in the ecliptic and not along the celestial equator, its right ascension does not increase uniformly.

Consider a first fictitious Sun travelling along the *ecliptic* with a constant speed and coinciding with the true Sun at the perigee and apogee (when the Earth is in perihelion and aphelion, respectively). Then consider a second fictitious Sun travelling along the *celestial equator* at a constant speed and coinciding with the first fictitious Sun at the equinoxes. This second fictitious Sun is the *mean Sun*, and by definition its right ascension increases at a uniform rate. [That is, there are no periodic terms, but its expression contains small secular terms in τ^2, τ^3, ...].

When the mean Sun crosses the observer's meridian, it is mean noon there. True noon is the instant when the true Sun crosses the meridian. The *equation of time* is the difference between apparent and mean time; or, in other words, it is the difference between the hour angles of the true Sun and the mean Sun.

Defined in this manner, the equation of time E, at a given instant, is given by

$$E = L_o - 0°.005\,7183 - \alpha + \Delta\psi \,.\, \cos \varepsilon \qquad (27.1)$$

In this formula, L_o is the Sun's mean longitude. According to the VSOP87 theory (see Chapter 31) we have, in degrees,

$$\begin{aligned} L_o = {} & 280.466\,4567 + 360\,007.698\,2779\,\tau \\ & + 0.030\,320\,28\,\tau^2 + \tau^3/49931 \\ & - \tau^4/15299 - \tau^5/1988\,000 \end{aligned} \qquad (27.2)$$

where τ is the time measured in Julian millennia (365 250 ephemeris days) from J2000.0 = JDE 2451 545.0. L_0 should be reduced to less than 360° by adding or subtracting a convenient multiple of 360°.

In the French almanacs and in older textbooks, the equation of time is defined with opposite sign, hence being equal to mean time minus apparent time.

In formula (27.1), the constant 0°.005 7183 is the sum of the mean value of the aberration in longitude (−20".49552) and the correction for reduction to the FK5 system (−0".09033); α is the apparent right ascension of the Sun, calculated by taking into account the aberration and the nutation. The quantity $\Delta\psi \cdot \cos\varepsilon$, where $\Delta\psi$ is the nutation in longitude and ε the obliquity of the ecliptic, is needed to refer the apparent right ascension of the Sun to the *mean* equinox of the date, as is the mean longitude L_0.

In formula (27.1), the quantities L_0, α and $\Delta\psi$ should be expressed in degrees. Then the equation of time E will be expressed in degrees too; it can be converted to minutes of time by multiplication by 4.

The equation of time E can be positive or negative. If $E > 0$, the true Sun crosses the observer's meridian before the mean Sun.

The equation of time is always less than 20 minutes in absolute value. If $|E|$ appears to be too large, add 24 hours to or subtract it from your result.

Example 27.a — Find the equation of time on 1992 October 13 at 0^h Dynamical Time.

This date corresponds to JDE = 2448 908.5, from which we deduce

$$\tau = \frac{\text{JDE} - 2451\,545.0}{365\,250} = -0.007\,218\,343\,600$$

$$L_0 = -2318°.192\,807 = +201°.807\,193$$

For the same instant we have, from Example 24.b,

$$\alpha = 198°.378\,178$$
$$\Delta\psi = +15''.908 = +0°.004\,419$$
$$\varepsilon = 23°.440\,1443$$

whence, by formula (27.1),

$$E = +3°.427\,351 = +13.70940 \text{ minutes} = +13^m42^s.6$$

Alternatively, the equation of time can be obtained, with somewhat less accuracy, by means of the following formula given by Smart [1] :

$$E = y \sin 2L_0 - 2e \sin M + 4ey \sin M \cos 2L_0 - \frac{1}{2}y^2 \sin 4L_0 - \frac{5}{4}e^2 \sin 2M \qquad (27.3)$$

where

$y = \tan^2 \frac{\varepsilon}{2}$, ε being the obliquity of the ecliptic,

L_0 = Sun's mean longitude,

e = eccentricity of the Earth's orbit,

M = Sun's mean anomaly.

The values of ε, L_0, e and M can be found by means of formulae (21.2), (27.2) or (24.2), (24.4), and (24.3), respectively.

The value of E given by formula (27.3) is expressed in radians. The result may be converted into degrees, and then into hours and decimals by division by 15.

Example 27.b — Find, once again, the value of the equation of time on 1992 October 13.0 TD = JDE 2448 908.5.

We find successively

$T = -0.072\,183\,436$ $\qquad e = 0.016\,711\,651$

$\varepsilon = 23°.44023$ $\qquad M = 278°.99396$

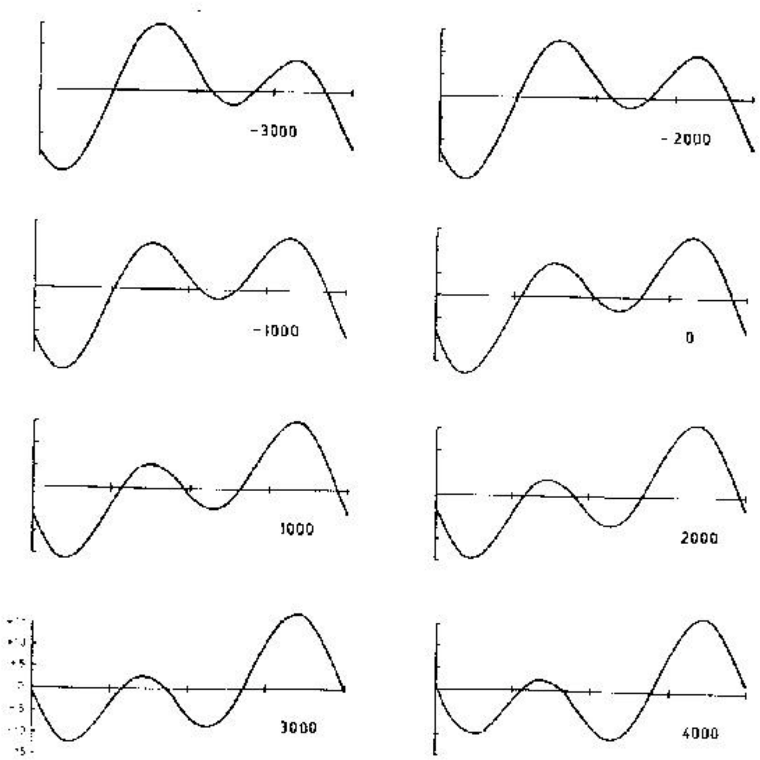
$L_0 = 201°.80720$ $\qquad y = 0.043\,0381$

Formula (27.3) then gives E = +0.059 825 557 radian
= +3.427 752 degrees
= +13 minutes 42.7 seconds

The curve representing the variation of the equation of time during the year is well-known and can be found in many astronomy books. Presently, this curve has a deep minimum near February 11, a high maximum near November 3, and a secondary maximum and minimum about May 14 and July 26, respectively.

However, the curve of the equation of time is gradually changing in the course of the centuries, because the obliquity of the ecliptic, the eccentricity of the Earth's orbit, and the longitude of the perihelion of this orbit are all slowly changing. The figure on the next page shows the curve of the equation of time at intervals of 1000 years, from 3000 B.C. to A.D. 4000. On the vertical scale, the tics are given at intervals of five minutes of time; the horizontal line represents the value E = zero. The tics on this horizontal line divide the year in four periods of three months each, beginning from January 1 at left. We see, for instance, that the minimum of February will be less deep in the future.

Between A.D. 1600 and 2100, the extreme values of the equation of



The curve of the equation of time, from −3000 to +4000

time vary as shown in Table 27.A. These are 'mean' values: the calculation is based on a non perturbed elliptical motion of the Earth, and the nutation has not been taken into account.

In A.D. 1246, when the Sun's perigee coincided with the winter solstice, the curve representing the annual variation of the equation of time was exactly symmetrical with respect to the zero-line: the minimum of February was exactly as deep as the height of the November maximum; and the smaller May maximum was exactly as high as the value of the July minimum — see the last line of the Table.

TABLE 27.A

The extreme values of the equation of time in modern times

Year	*Minimum of February*	*Maximum of May*	*Minimum of July*	*Maximum of November*
	m s	*m s*	*m s*	*m s*
1600	−15 01	+4 19	−5 40	+16 03
1700	−14 50	+4 09	−5 53	+16 09
1800	−14 38	+3 59	−6 05	+16 15
1900	−14 27	+3 50	−6 18	+16 20
2000	−14 15	+3 41	−6 31	+16 25
2100	−14 03	+3 32	−6 44	+16 30
1246	−15 39	+4 58	−4 58	+15 39

Reference

1. W.M. Smart, *Text-Book on Spherical Astronomy*; Cambridge (Engl.), University Press (1956); page 149.