Chapter 7

Finding Time *and* Longitude by Lunar Observations

In navigation, time and longitude are interdependent. Determining one's longitude requires knowledge of the exact time and vice versa. This fundamental problem remained unsolved for many centuries, and old-time navigators were restricted to latitude sailing, i. e., traveling along a chosen parallel of latitude. As a result, the time of arrival could only be estimated, and many ships ran ashore at nighttime or when visibility was poor due to bad weather. Before the invention of the first reliable chronometer by John Harrison, many attempts were made to use astronomical events, e.g., solar eclipses and occultations of Jupiter moons as time marks. Although these methods were fairly accurate, many of them were impracticable at sea. Among the astronomical methods, deriving the time from **lunar distances** deserves special attention. After refined methods for the accurate prediction of the moon's apparent position became available in the 18th century, the angular distance of the moon from a chosen body, preferably one in or near the moon's path, compared with the predicted angular distance, could be utilized to determine the error of a less accurate timepiece at certain intervals in order to calibrate the instrument. This is possible since the moon's apparent position with respect to other heavenly bodies varies comparatively rapidly. The procedure was still in use in the second half of the 19th century due to the high price of precision chronometers [4].

In practice, the method of lunar distances was very complicated. It required measuring the angular distance between the moon's illuminated limb and a chosen body and the altitudes of the moon and said body at approximately the same time. This was usually done by three or four observers. Then a number of complex calculations had to be performed in order to convert the topocentric angular distance to the geocentric angular distance. These calculations included corrections for refraction (both bodies) as well as parallax and augmented semidiameter of the moon. Reportedly, it took several hours to complete this procedure called „clearing the lunar distance“.

We will not discuss the traditional method of lunar distances in detail here although it is an intellectual challenge. Instead, we will develop a less complicated way to derive the chronometer error from lunar observations using the well-known sight reduction formulas from chapter 4 and the equation of equal altitudes described in chapter 6. This method, too, uses the hourly variation in the siderial hour angle of the moon as a time standard.

GHA_{Aries} increases by 902.46 arcminutes per hour. Since siderial hour angles and declinations of fixed stars change very slowly, each stellar circle of equal altitude travels westward at about the same rate. Accordingly, a chronometer error of +1h (chronometer fast) displaces any **stellar line of position** as well as any **stellar fix** 902.46 arcminutes to the west.

The GHA of the moon increases only at a rate of 859.0+*v* arcminutes per hour. Accordingly, a chronometer error of +1h displaces any **lunar line of position** 859.0+*v* arcminutes to the west, provided the declination of the moon is constant. The small quantity *v*, the variable excess over the adopted minimum value of 859.0 arcminutes per hour, is tabulated on the daily pages of the Nautical Almanac.

As a result of the different hourly changes of GHA_{Aries} and the GHA of the moon, the siderial hour angle of the moon changes at a rate of *v*-43.46 arcminutes per hour (retrograde motion):

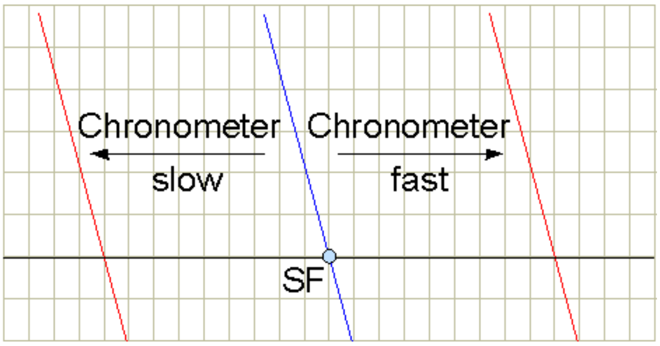
$$\Delta SHA_{Moon}\left[{}'\right] = \left(859.0 + v - 902.46\right)\cdot \Delta T\left[h\right] = \left(v - 43.46\right)\cdot \Delta T\left[h\right]$$

If we neglect the geometrical errors of the intercept method, a lunar line of position passes exactly through a fix derived from the altitudes of two or more stars if our chronometer shows the accurate time, provided the observations are error-free and the observer remains stationary. The lunar LoP and the stellar fix drift apart as the chronometer error increases.

Constant declination of the moon provided, the angular distance of the lunar LoP from the stellar fix, measured along the parallel of latitude going through the fix, equals the change in the siderial hour angle of the moon, ΔSHA, during the time interval ΔT, the chronometer error.

As demonstrated in *Fig. 7-1*, the lunar LoP appears eastward from the stellar fix, SF, (ΔSHA negative) if the chronometer error, ΔT, is positive (chronometer fast). The lunar LoP appears westward from stellar fix (ΔSHA positive) if the chronometer error is negative (chronometer slow).

Fig. 7-1



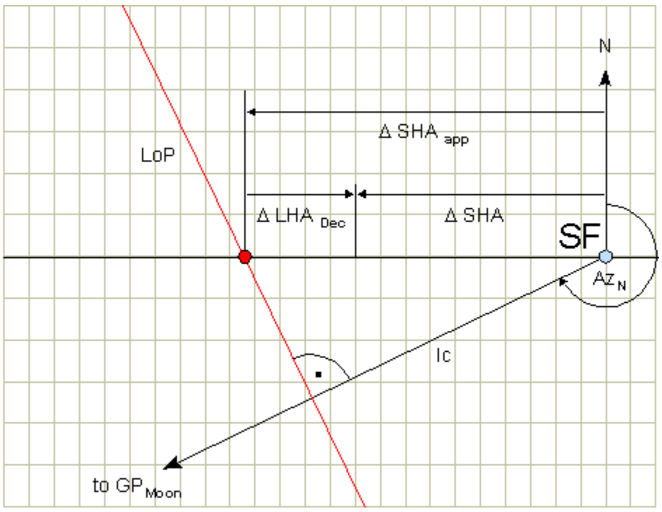
We begin with measuring the altitudes of two (or more) stars and the moon and applying the usual corrections. At each observation, we note the time as indicated by our chronometer. Measurements and altitude corrections should be made with greatest care and accuracy. Intermediate results should not be rounded but noted to the last digit. The altitude corrections for the moon should include the small correction for the oblateness of the earth (see chapter 2).

Next, we derive a fix from the star observations using the intercept method (see chapter 4). To reduce the geometrical error caused by the curvatures of the position lines, we calculate as many iteration cycles as necessary to obtain a constant position (usually 2 or 3). Of course, our stellar fix has an unknown longitude error. This is not a problem since we only evaluate differences in longitude. The obtained latitude, however, is accurate. We will need it during the further course of our calculations.

We use the fix thus obtained as our assumed position to derive the lunar line of position. Again, we apply the sight reduction formulas for the intercept method.

Fig. 7-2 shows a graphic plot of the lunar line of position. The point where the lunar LoP intersects the parallel of latitude going through the stellar fix, SF, marks ΔSHA_{app}. The latter is the westward or eastward shift of the lunar LoP with respect to SF caused by the combined effects of the chronometer error, ΔT, and the corresponding change in the declination of the moon, ΔDec.

Fig. 7-2



$$\Delta SHA_{app}['] = -\frac{Ic[nm]}{\sin Az_N \cdot \cos Lat}$$

The above formula is an approximation neglecting the curvature of the lunar LoP. The resulting error is not significant if the azimuth is in the area of 90° or 270° (±30°) and if the altitude of the moon is not too high (< ~70°). Usually, the influence of observation errors on the final result is many times greater.

ΔSHA equals ΔSHA_{app} only if the moon's declination is constant during the interval ΔT. Usually, the moon's declination changes rapidly. The quantity d tabulated on the daily pages of the Nautical Almanac is the hourly change in declination measured in arcminutes. We remember that d can be positive or negative, depending on the current trend of declination. The change in Dec during the interval ΔT is:

$$\Delta Dec\,['] = d \cdot \Delta T\,[h]$$

ΔDec is equivalent to a change in the local hour angle, ΔLHA_{Dec}, if we consider equal altitudes of the moon. ΔLHA_{Dec} can be positive or negative. We can calculate ΔLHA_{Dec} with the **equation of equal altitudes** (see chapter 6):

$$\Delta LHA_{Dec} = f \cdot \Delta Dec \qquad f = \left(\frac{\tan Lat}{\sin LHA} - \frac{\tan Dec}{\tan LHA} \right)$$

LHA is the algebraic sum of the GHA of the moon and the longitude of the stellar fix.

Thus, we get:

$$\Delta LHA_{Dec}\,['] = f \cdot d \cdot \Delta T\,[h]$$

ΔSHA is the algebraic sum of ΔSHA_{app} and ΔLHA_{Dec}:

$$\Delta SHA = \Delta SHA_{app} + \Delta LHA_{Dec}$$

Combining the above formulas, we have:

$$(v - 43.46) \cdot \Delta T\,[h] = f \cdot d \cdot \Delta T\,[h] - \frac{Ic\,[nm]}{\sin Az_N \cdot \cos Lat}$$

Solving the equation for ΔT (in seconds of time), we get the chronometer error:

$$\Delta T\,[s] = 3600 \cdot \frac{Ic\,[nm]}{(43.46 - v + f \cdot d) \cdot \sin Az_N \cdot \cos Lat}$$

According to the hourly variation of GHA_{Aries}, our improved longitude is:

$$Lon_{improved} = Lon + C \qquad C\,['] = 0.25068 \cdot \Delta T\,[s]$$

Lon is the raw longitude obtained by our observations of stars.

One should have no illusions about the accuracy of longitudes obtained by lunar distances and related methods since small observation errors result in large errors in time and longitude. This is due to the fact that siderial hour angle and declination of the moon change slowly compared with the rate of change of GHA which is the basis for usual sight reduction procedures. Longitude errors of 1° were considered as normal in the days of lunar distances when nothing better was available.

The above method works best if the azimuth of the moon is 90° or 270°. In practice, an azimuth of 90°±30° or 270°±30° is acceptable. The optimum altitude of the moon is a trade-off between refraction errors (low altitude) and the curvature of the LoP (high altitudes). Therefore, medium altitudes (20°-60°) are preferred. The accuracy of the method decreases with increasing latitude. Therefore, it should not be used in polar regions. In any case, the observer has to be stationary. If the Observer's position changes during the observations, intolerable errors will result. The overall error can be reduced by multiple observations. It is therefore recommended to make a series of observations (see chapter 16).

Compared with the traditional method of lunar distances, the above procedure has not only the advantage that the required formulas are much simpler but also that it can be managed by one person since it is not necessary to make the observations at the same time. On land, relatively accurate results can be obtained when using a theodolite to measure the altitudes. In this case, the residual error in longitude usually does not exceed a few arcminutes.